Understanding how students learn mathematics and knowing which instructional methods are likely to be successful should inform instructional practice. Effective mathematics instruction requires a variety of resources, materials, technology, and delivery formats that take into account students’ different learning styles and instructors’ different teaching styles. Every teaching activity should promote active learning and be guided by informed decision-making.

**Implementation Standard: Instruction**

Mathematics faculty will use a variety of teaching strategies that reflect the results of research to enhance student learning.

**Teaching Styles**

“Teaching style” refers to an instructor’s content-independent, persistent qualities, attitudes, and traits. It is directly linked to the instructor’s educational philosophy and a subset of the instructor’s life philosophy. Teaching styles may be student-centered or teacher-centered or any combination of the two. In a student-centered style, the instructor expects the student to take responsibility for learning and the student trusts the instructor will help. Student-centered models involving a constructivist approach and active learning have been shown to be successful for students with non-traditional learning styles. In a teacher-centered style, the instructor organizes the transmission of content knowledge to the learner, acts as a performance assessor, and provides opportunities for the learner to practice independently. Understanding where an instructor fits in the continuum of styles and using that information to make informed decisions about instruction is one of the steps in becoming a professional educator, rather than simply a teacher. Regardless of which teaching style is preferred, using multiple instructional strategies is recommended to address as many individual learning styles (see Chapter 4).
Mathematics instruction may be improved through sensitivity to learning styles. However, matching teaching style to learning style has limitations and does not necessarily guarantee greater student achievement.\(^8\) One meta-study involving children concluded that there was little effect on learning when a teaching strategy was chosen specifically to respond to a student’s particular learning style.\(^9\) In other research involving college students, matching students’ learning styles with a variety of instructional strategies showed some success, particularly with new students and poorly prepared students, where most attrition occurs.\(^10\) To become more aware of the kinds of methods an instructor is selecting, the instructor can code lesson plans by placing a V next to visual activities, an R next to reflective activities, etc. “If the coding system is used on a regular basis, it becomes very natural to think in terms of being inclusive, or providing the setting and the activities by which all learners can find some portion of the class that particularly appeals to them.”\(^11\) In selecting instructional strategies, the goal should be to find the best instructional strategy for the particular content being taught\(^12\) and for the particular level of cognitive development of the students in the class.

### Table 6  Student-Centered and Teacher-Centered Teaching Styles\(^3\)

<table>
<thead>
<tr>
<th>Student-Centered</th>
<th>Teacher-Centered</th>
</tr>
</thead>
<tbody>
<tr>
<td>Knowledge is constructed by students gathering and synthesizing information.</td>
<td>Knowledge is transmitted from instructor to students.</td>
</tr>
<tr>
<td>Emphasis is on using knowledge to address problems students will experience outside of the classroom.</td>
<td>Emphasis is on the acquisition of knowledge, sometimes outside the context in which it will be used.</td>
</tr>
<tr>
<td>The instructor’s role is coach, facilitator, and mentor; students and instructor assess learning performance together.</td>
<td>The instructor’s role is primarily as an information-giver and performance assessor.</td>
</tr>
<tr>
<td>Assessment is used to diagnose learning problems, in addition to evaluating learning outcomes.</td>
<td>Assessment is used to evaluate learning outcomes.</td>
</tr>
<tr>
<td>Teaching and assessment of learning are intertwined.</td>
<td>Teaching and assessment of learning are separate.</td>
</tr>
<tr>
<td>The learning culture is cooperative and collaborative.</td>
<td>The learning culture is competitive and individualistic.</td>
</tr>
<tr>
<td>Instructors and students learn together.</td>
<td>Students are viewed as learners.</td>
</tr>
</tbody>
</table>

The diverse needs of students in any mathematics course call for diverse approaches to instruction. While knowledge of content is essential in teaching any discipline, effective teaching is the result of integrating content and pedagogical knowledge.\(^4\) Effective instructors venture outside their comfort zone, take risks, grow in their teaching, and model the behavior they expect from their students.\(^5\) The instructional methods chosen can encourage or inhibit students’ ability to demonstrate their mathematical knowledge and take ownership for creating and understanding mathematical ideas.\(^6\) Certain teacher practices such as providing students with frequent opportunities to experience problem solving in a variety of contexts, allowing adequate time for solving problems, devoting time specifically for planning, monitoring, and reflecting on progress and the answer, have been shown to help students improve problem-solving skills. “A variety of instructional strategies are necessary to respond to as many different learning and teaching styles as possible.”\(^7\)

Mathematics instruction may be improved through sensitivity to learning styles. However, matching teaching style to learning style has limitations and does not necessarily guarantee greater student achievement.\(^8\) One meta-study involving children concluded that there was little effect on learning when a teaching strategy was chosen specifically to respond to a student’s particular learning style.\(^9\) In other research involving college students, matching students’ learning styles with a variety of instructional strategies showed some success, particularly with new students and poorly prepared students, where most attrition occurs.\(^10\) To become more aware of the kinds of methods an instructor is selecting, the instructor can code lesson plans by placing a V next to visual activities, an R next to reflective activities, etc. “If the coding system is used on a regular basis, it becomes very natural to think in terms of being inclusive, or providing the setting and the activities by which all learners can find some portion of the class that particularly appeals to them.”\(^11\) In selecting instructional strategies, the goal should be to find the best instructional strategy for the particular content being taught\(^12\) and for the particular level of cognitive development of the students in the class.

**Implementation recommendation:** Faculty will recognize their own teaching style(s), reflect on the implications of their style on their students’ learning styles, and use that knowledge and other research to make informed decisions about the selection of multiple instructional designs and their classroom management.
Actions to support this recommendation

Faculty actions:

✦ respect diverse talents and ways of learning and teaching
✦ design and use a variety of classroom activities, assignments, and assessments to address multiple learning styles
✦ encourage student-faculty contact
✦ provide students with prompt feedback
✦ be attentive, expressive, enthusiastic, clear, and organized
✦ present multiple representations of mathematical concepts.

Active Student Learning

Many college students today have grown up with technology and as a result, they process information differently than students who did not. This characteristic has implications for mathematics classes, in general, not just courses using calculators or computers. When planning classroom activities, instructors need to acknowledge that their students are

✦ are intuitive visual communicators
✦ learn better by making observations or through discovery; they like doing things, not just thinking or talking about things
✦ can shift their attention rapidly from one task to another
✦ are able to respond quickly and expect rapid responses in return
✦ prefer to work in teams.

For today’s students, learning is participatory—knowing depends on practice and participation.

New findings in the science of learning suggest that learning requires some sort of cognitive restructuring of material by the learner. “Transfer can be improved by helping students become more aware of themselves as learners who actively monitor their learning strategies and resources and assess their readiness for particular tests and performances.” Therefore, instructors need to draw out and work with pre-existing understandings students bring with them.

Collaborative and cooperative learning are examples of strategies to encourage active student learning. Collaborative learning is defined strictly as an unstructured process in which participants define problems, develop procedures, and produce socially constructed knowledge. Collaborative learning develops social skills, promotes student-student and student-faculty interactions and builds self-esteem in students as it develops mathematics skills and knowledge. A shy student, when placed in a small group, still benefits from collaborative learning by observing strategies used by his/her peers. One of the most effective means of restructuring is by explaining the material to someone else. Cooperative learning is a structured, systematic instructional strategy in which groups work toward a common goal. In practice, most group learning activities are a mix of cooperative and collaborative learning. The goal of both collaborative and cooperative learning is for students to be actively involved in learning activities that lead to a rich understanding of the course content.

Interactive lecturing, question posing, and inquiry-based strategies have also been shown to increase students’ active learning, interest, and curiosity. Designing questions and problems that are challenging yet within student’s cognitive development and giving students time to respond are critically important for...
these instructional methods to be effective. Discovery-based learning utilizing carefully designed questions and activities can lead students to connect new knowledge to previous knowledge. Additional questions may be necessary to facilitate knowledge construction for some students. Allowing a learner to grapple with the logic behind a rule and then apply it may be frustrating, but develops understanding of concepts.¹⁹

The appropriate use of writing in mathematics courses is another strategy that can play an important role in the process of internalizing mathematical procedures, understanding the relationships of mathematical concepts, and synthesizing different mathematical components into a coherent schema.²⁰ Writing “is an essential activity to create order from chaos, sense from nonsense, meaning from confusion: as such it is the heart of creative learning in both the arts and sciences.”²¹

Active learning occurs in many formats such as collaborative learning, discovery-based learning, interactive lecturing and question posing, and writing. Whichever format is chosen, the goal of the activity should be to enhance conceptual understanding.

Table 7  Instructional Strategies That Promote Active Learning

<table>
<thead>
<tr>
<th>Strategies for Faculty</th>
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<tbody>
<tr>
<td>Collaborative/Cooperative Learning</td>
</tr>
<tr>
<td>• Provide pairs or small groups of students the opportunity to play a mathematical game, solve a textbook problem, explain, engage in a research project, interpret a graph to another student, review another student’s work, discuss the correct solution, review and critique a video.</td>
</tr>
<tr>
<td>• Assign Internet group projects (in or out of class), technology activities or activities where measurements are taken and analyzed.</td>
</tr>
<tr>
<td>• Facilitate informal study groups outside of class.</td>
</tr>
<tr>
<td>• Ask students to work in pairs on a homework problem before class begins.</td>
</tr>
<tr>
<td>• Provide a location for group review for an upcoming test.</td>
</tr>
<tr>
<td>Discovery-based Learning</td>
</tr>
<tr>
<td>• Use student answers to guide classroom discussion.</td>
</tr>
<tr>
<td>• Present examples that lead to patterns, which form the basis of mathematical rules.</td>
</tr>
<tr>
<td>• Ask students to discover concepts and patterns.</td>
</tr>
<tr>
<td>Interactive Lecturing and Question-Posing</td>
</tr>
<tr>
<td>• Ask students to raise their hand and select the third or fourth hand for a response.</td>
</tr>
<tr>
<td>• Ask all students to solve a problem and then compare their processes and/or answer with a student nearby.</td>
</tr>
<tr>
<td>• Respond to any answer given (right or wrong) and discuss the logical ramifications of trying to solve the problem using alternative methods.</td>
</tr>
<tr>
<td>• Ask questions to guide students to solutions to problems.</td>
</tr>
<tr>
<td>• Engage students in activities that lead them to develop conceptual understandings.</td>
</tr>
<tr>
<td>• Ask students to write questions that require explanations.</td>
</tr>
<tr>
<td>Writing</td>
</tr>
<tr>
<td>• Ask students to write to an absent student explaining the most important mathematics concept from the day’s lesson.</td>
</tr>
<tr>
<td>• Ask students at the beginning of the class to write about what they learned from doing homework.</td>
</tr>
<tr>
<td>• Encourage students to take notes and/or keep track of important mathematics ideas and problem statements by writing a checklist on the board or disseminating a fact sheet.</td>
</tr>
<tr>
<td>• Ask members of a group to describe how to perform mathematical processes integral to the mathematical concept.</td>
</tr>
<tr>
<td>• Include a writing component in homework, student projects, investigations, and explorations of mathematical concepts.</td>
</tr>
</tbody>
</table>
Implementation recommendation: Faculty will design and implement instructional activities that actively engage students in the learning of mathematics.

Actions to support this recommendation

Faculty actions:

✦ design and implement multiple instructional approaches that promote active student participation in the learning process
✦ formulate activities that require students to memorize, comprehend, apply, analyze, and synthesize mathematical concepts
✦ allow discovery-based and thought-provoking questions and activities to guide classroom discussions
✦ provide opportunities for and encourage students to think, reflect, discuss, and write about mathematical ideas and concepts.

Departmental/institutional actions:

✦ provide faculty with appropriate facilities and technology to design and implement instructional strategies that engage students actively
✦ provide faculty with professional development opportunities to improve their instructional strategies.

Teaching with Technology

Advances in hardware and software technology have had a tremendous impact on our lives. The infusion of technology into education presents interesting opportunities for teaching and learning, especially in mathematics. Technology changes not only how mathematics is taught, but also when and what mathematics is taught.

The definition and use of technology in mathematics education is constantly evolving. Technology may refer to the use of graphing calculators, student response systems, online laboratories, simulations and visualizations, mathematical software, spreadsheets, multimedia, computers or the Internet, and other innovations yet to be discovered. Technology can be used to learn mathematics, to do mathematics, and to communicate mathematical information and ideas. The Internet hosts a wealth of mathematical materials that are easily accessible through the use of search engines, creating additional avenues to enhance teaching and facilitate learning. Outside of class, students and faculty can pose problems and offer solutions through e-mail, chat rooms, or websites. Technology provides opportunities for educators to develop and nurture learning communities, embrace collaboration, provide community-based learning, and address diverse learning styles of students and teaching styles of teachers.22

Faculty using technology should answer the question: How can the use of technology facilitate learning that is durable, has substance, is engaging to students, and provides mathematical insights through a high level of understanding of the mathematics being taught? They should expose students to a variety of technology options appropriate for solving different types of problems and guide them in making appropriate choices about the use of technology for problem solving.

Technology enhances the learning of mathematics, when used appropriately as a tool, to achieve the following:

✦ explore new concepts and discover patterns
✦ examine, organize, analyze, and visualize real-world data
✦ develop understandings of mathematical ideas
The integration of appropriately used technology can enhance student understanding of mathematics through pattern recognition, connections, and dynamic visualizations. Electronic teaching activities can attract attention to the mathematics to be learned and promote the use of multiple methods. Learning can be enhanced with electronic questioning that engages students with technology in small groups and facilitates skills development through guided-discovery exercise sets. Using electronic devices for communication, all students can answer mathematics questions posed in class, while allowing the technology to perform routine and repeated calculations. The use of technology should not be used as a substitute for an understanding of and mastery of basic mathematical skills. Technology should be used to enhance conceptual understanding, while simultaneously improving performance in basic skills.

The integration of appropriately used technology can enhance student understanding of mathematics through pattern recognition, connections, and dynamic visualizations. Electronic teaching activities can attract attention to the mathematics to be learned and promote the use of multiple methods. Learning can be enhanced with electronic questioning that engages students with technology in small groups and facilitates skills development through guided-discovery exercise sets. Using electronic devices for communication, all students can answer mathematics questions posed in class, while allowing the technology to perform routine and repeated calculations. The use of technology should not be used as a substitute for an understanding of and mastery of basic mathematical skills. Technology should be used to enhance conceptual understanding, while simultaneously improving performance in basic skills.

Students will be expected to use technology to do the following:

- enhance their understanding of mathematics
- discover mathematical concepts and patterns
- perform mathematical tasks
- visualize different representations of the same mathematical concept
- formulate and test conjectures about mathematical concepts and procedural rules
- communicate mathematical information and ideas.

Implementation recommendation: Faculty will integrate technology appropriately into their teaching to enhance students’ understanding of mathematical concepts and skills.
Actions to support this recommendation

Faculty actions:
✦ integrate technology into their teaching of mathematics
✦ use technology tools for assessment that are aligned with instruction
✦ align technology platforms with those familiar to students, required for future courses, and/or necessary in their future careers.

Departmental/institutional actions:
✦ provide technology with options for interactivity between students and faculty supporting classroom activities and student learning of mathematics
✦ provide technology for students to learn and faculty to teach mathematics courses.

Distance Learning

Distance learning, or learning at a distance, involves alternative delivery formats where the instructor and the student are separated either by time, location, or both. Distance learning courses may be delivered synchronously (in real-time), asynchronously (in delayed time), or a combination of both (hybrid) through written correspondence, text, graphics, various digital media, interactive television, videoconferencing, or online. Instruction can involve a variety of distance learning delivery formats:
✦ Web-displayed, the traditional classroom with the syllabus and some readings and assignments online
✦ Web-enhanced, the traditional classroom with some lectures and interactions online
✦ hybrid/blended, with both the Web-displayed and Web-enhanced components and increased online content and interaction
✦ completely online.

Faculty across the nation have developed and implemented distance learning courses and programs in an effort to provide alternative delivery formats, flexibility in course scheduling, and accessibility for students. Yearly reports from the National Center for Educational Statistics show an increase in the number of courses offering some degree of technology-based delivery.25 Education at a distance allows students with work and personal commitments to take courses at their convenience and participate in a dynamic and interactive process for student learning.26

The extraordinary growth of technology-mediated mathematics courses in higher education has created new challenges. The pressure to expand the number of courses taught at a distance must be balanced with sound strategies for assessment to assure quality. Students and faculty face new challenges with regard to communicating mathematics through these different media. Faculty face new challenges as they attempt to reach the same goals for problem solving, critical thinking, collaboration, communication, content, and assessment as they have in a seated class. These challenges require that professional organizations provide faculty with opportunities to share experiences and learn from one another through conferences, workshops, and by participating in electronic discussions sponsored by the organizations.

Mathematics distance education programs must be carefully planned so they effectively deliver distance education and achieve the same goals and objectives and student learning outcomes as the same course taught in a more traditional classroom. Developing interactive distance learning tools to promote active learning in mathematics requires appropriate infrastructure, professional development, and
resources. Policy and accrediting agencies have outlined benchmarks and factors such as institutional context and commitment, curriculum and course development, teaching and learning, faculty support, student support, and evaluation and assessment to ensure excellence in distance education and reflect quality.27

The capacity of the college media and computing services to support a distance learning program is an important consideration. Access and equity must be assured for all students. Student outcomes, expectations, and responsibilities must be clearly outlined and communicated. Orientation and advising, specifically related to distance learning, must be provided to students. Students must be aware of the personal attributes required to be successful in courses that have an alternative delivery format. Faculty and departments should develop strategies that help students make wise decisions about which delivery format is most likely to result in success. In addition, students must have easy access to the hardware and software required in the course. Faculty should consider requiring students to use the platform on which the course will be delivered as a prerequisite to registration. Essential components of a distance learning course include the following:28

✦ a documented technology plan that is in place and operational
✦ minimum standards that guide course development, design, and delivery based on learning outcomes, not on the availability of existing technology
✦ students actively engaged in analysis, synthesis, and evaluation of the course
✦ periodic reviews of learning outcomes and instructional materials
✦ necessary mathematical software and hardware available to all students enrolled in distance learning mathematics courses
✦ focus on goals, not on delivery format
✦ models of course development, instruction, and assessment that address sustainability and course enhancement beyond the involvement of the original developer(s).

The Implementation Cycle of Beyond Crossroads can be helpful in defining goals and objectives, assessing effectiveness, and making improvements in distance learning courses on a continuing basis.

**Students will be expected to do the following:**

✦ engage in distance learning orientation and advising for distance learning mathematics courses
✦ interact with faculty and students in the distance learning mathematics courses in which they are enrolled
✦ assume responsibility for their learning in an alternative delivery format.

**Implementation recommendation:** Faculty will utilize effective instructional design practices when developing and implementing distance learning courses in mathematics.

**Actions to support this recommendation**

Faculty actions:

✦ select technology that is accessible to students enrolled in their distance learning mathematics course
✦ advise students on the expectations of their distance learning mathematics course and orient them to the distance learning environment of their course
✦ provide students with course information outlining course objectives, concepts, ideas, and learning outcomes for their distance learning mathematics course
✦ engage in ongoing professional development to enhance their mathematics course presentation and support their teaching practice in the distance learning environment
assure that learning outcomes in mathematics distance learning sections are consistent with those of similar mathematics courses taught in classrooms.

**Departmental/institutional actions:**

- provide and adequately maintain the infrastructure and resources to support the development and teaching of distance learning courses in mathematics
- codify distance learning policies and procedures for mathematics students, faculty, and support staff and develop appropriate instruments for student and peer evaluation of distance learning mathematics courses
- provide distance learning orientation, advising, hands-on training, and support services to students registered for or intending to register for distance learning mathematics courses
- provide professional development to support mathematics faculty developing and teaching distance learning courses
- establish a model for distance learning course development, instruction, and assessment that will be sustainable after the tenure of the developer(s).

**Conclusion**

The teaching of mathematics should be guided by research in learning theory, should incorporate active student participation, and utilize multiple instructional strategies to maximize student success in mathematics. The use of technology in a mathematics classroom should help students become active learners and develop a deep understanding and profound appreciation of mathematics. Instructional strategies and delivery formats should be designed to respond to different learning and teaching styles, to use technology appropriately to enhance learning, to provide opportunities of access and academic success for all students, and to facilitate avenues for collaboration and cooperation.


