Chapter 1

Beyond Crossroads: Implementing Mathematics Standards in the First Two Years of College

Shaping the Vision

In 1995, the American Mathematical Association of Two-Year Colleges (AMATYC) released its first standards document, *Crossroads in Mathematics*.[1] *Crossroads in Mathematics* emphasized desired modes of student thinking and guidelines for selecting content and instructional strategies. The purposes of AMATYC's second standards document, *Beyond Crossroads*, are to renew and to extend the goals, principles, and standards set forth in *Crossroads* and to continue the call for their implementation. *Beyond Crossroads* presents a renewed vision for mathematics courses offered in the first two years of college with an additional set of standards, called Implementation Standards, which focus on student learning and the learning environment, assessment of student learning, curriculum and program development, instruction, and professionalism.

Beyond Crossroads is intended to stimulate faculty, departments, and institutions to examine, assess, and improve every component of mathematics education in the first two years of college. The varied challenges for full-time and adjunct faculty are fully acknowledged. Faculty need and deserve the necessary facilities, equipment, and professional development opportunities essential for performing their teaching responsibilities. These standards, recommendations, and action items are not intended to be a prescription for action used identically by each faculty member, department, or institution. Rather, they are to be used as a starting point for dialogue, reflection, experimentation, evaluation, and continuous improvement. Used in this way, this document can guide professionals toward standards-based mathematics education that promotes continuous professional growth and helps students maximize their potential in every college mathematics course.

The ultimate goals of this document are to improve mathematics education and to encourage more students to study mathematics.

Crossroads in Mathematics: Standards for Introductory College Mathematics Before Calculus, AMATYC, 1995, p. 1

What is a mathematics standard? Mathematics faculty, administrators, mathematics education researchers, policy makers, politicians, and parents continue to engage in dialogue on the meaning and role of standards. The words "educational standard" can have any of the following meanings depending on the audience and the purpose for which the standard is developed:[2]

- a vision of ideal practice
- essential knowledge in a field
- descriptors of student performance
- guides to align system components
- measurable goals for student learning

- clear curricular goals for instructors
- guides in measurement and accountability systems
- mechanisms that encourage dialogue and consensus building.

"Nationally developed standards in mathematics, science, and technology represent a set of fundamental changes in the way these subjects have traditionally been taught..."[3] The Standards for Intellectual Development, Content, and Pedagogy of *Crossroads in Mathematics* focused on content and instruction, describing what students should know and be able to do in mathematics and outlining pedagogical principles. To accomplish those standards, additional levels of support are needed.[4] The Implementation Standards of *Beyond Crossroads* address the key elements of student expectations, facilities, human resources, materials, curriculum design, instructional delivery formats, and professional development to achieve the standards of *Crossroads in Mathematics*.

One of the goals of *Beyond Crossroads* is to clarify issues, interpret, and translate research to bring standards-based mathematics instruction into practice. Two definitions from the literature of standards-based education have been adopted:

- Standards-based education entails implementing strategies (related to learning, assessment, curricula, teaching, and professionalism) and policies of what students should know and be able to do.[5]
- Standards-based education implies a greater coherence, or alignment, among the parts of the educational systems.[6] The assumption is that components/strategies that are aligned are more likely to be successful.

To accomplish this alignment, *Beyond Crossroads* has integrated recommendations from AMATYC position statements and related mathematics organizations: *Principles and Standards for School Mathematics* of the National Council of Teachers of Mathematics (NCTM)[7] and *Undergraduate Programs and Courses in the Mathematics Sciences: CUPM Curriculum Guide 2004*, a report of the Committee on the Undergraduate Program in Mathematics of the Mathematical Association of America (MAA).[8]

The Implementation Standards of *Beyond Crossroads* are first presented in Chapter 3. Then, each of Chapters 4 through 8 focuses on one Implementation Standard. Sections within those chapters address key issues for implementing standards-based mathematics education and include the following:

- supporting research on key issues
- lists of expectations of students to be communicated by faculty to help students set ambitious goals for themselves, accept responsibility for their own learning, and achieve greater success in their mathematics courses
- implementation recommendations
- action items for faculty offer guidance for designing curricula, choosing instructional strategies, and continuing to develop and contribute to the profession throughout their careers
- action items for departments and institutions outline ways to improve the learning environment, develop curricula, and support faculty.

The focus of the document is two-year college mathematics education and lower division mathematics education at four-year colleges and universities with characteristics similar to two-year colleges. The primary audience for *Beyond Crossroads* is two-year college mathematics faculty.[i] Since faculty who teach lower division mathematics at four-year colleges and universities are faced with similar issues as two-year college faculty, they are also an important audience. Additional audiences include college administrators, K–12 teachers, policy makers, government agencies, professional societies, publishers,

[i] Throughout this document, the term *faculty* will refer to both full-time and adjunct (part-time) professional teaching staff.

and funding agencies. Individuals, organizations, and businesses outside of education are called upon in *Beyond Crossroads* to collaborate with educators and each other, to improve college mathematics programs, and to respond to the needs of the mathematics community. This document serves as a call to action for all stakeholders to work together to improve student success in mathematics courses and programs in the first two years of college, with particular emphasis on the two-year college student, faculty, and institution characteristics.

I advise my students to listen carefully the moment they decide to take no more mathematics courses. They might be able to hear the sound of closing doors.

James Caballero, Everybody a Mathematician? CAIP Quarterly, Fall 1989, 2(2), p. 2

Distinctive Characteristics of Two-Year Colleges, Students, and Faculty

The distinctive characteristics of two-year colleges, students, and faculty make a compelling case for the development and implementation of distinguishing standards for mathematics in the first two years of college. In the century since its inception,[9] the two-year college has grown to offer a wide range of transfer, technical, and career-specific courses and programs to a diverse student population.

There were more than 1,150 two-year colleges serving 10.1 million students, with 6.6 million enrolled in credit classes in the year 2005. A two-year college was within commuting distance of nearly every person in the United States. In the academic year 2001–2002, 53 percent of all undergraduate students in the United States were enrolled at two-year colleges.[10] Most two-year colleges offered the following courses, programs, and services:

- open-door admission
- extensive developmental education programs
- a wide range of lower-division undergraduate courses that transfer to four-year colleges
- two-year associate degree programs in the arts and the sciences
- two-year associate degree programs in technical careers designed to meet critical local economic needs
- certificate programs for training in entry-level job skills
- short-term job training and continuing education related to local employment and economic growth
- adult literacy education and high school GED diplomas
- mandatory placement testing in mathematics for 98% of first-time mathematics students with mandatory placement at two-thirds of the colleges
- mathematics tutoring labs, staffed by full-time or adjunct faculty, paraprofessionals, or peer tutors.

In the academic year 2001–2002, two-year college students had these characteristics:[11]

- the average age was 29; 36 percent were 18–21 years old; 15 percent were 40 years or older
- 58 percent were women and thirty-three percent were minority students (black, native American, Asian/Pacific Islander, Hispanic)
- 61 percent of all students took a part-time course load
- 80 percent were employed with 41 percent employed full-time
- many two-year college students were involved in a career change, had not attended school in several years, and were commuters.

Approximately 1.3 million students enrolled in the following mathematics courses at two-year colleges in fall 2005.

Table 1 Percent of Students Enrolled in Mathematics Courses at Two-Year Colleges in 2005[12]

Mathematics Course	Percent
Developmental mathematics (precollege)	57
Precalculus	19
Calculus	6
Statistics	7
Other mathematics courses*	11

*These include linear algebra, probability, discrete mathematics, finite mathematics, mathematics for liberal arts, mathematics for elementary school teachers, technical mathematics, and computing.

The 8,793 full-time permanent faculty teaching mathematics in two-year colleges in the year 2005 had the following characteristics:[13]

- 44 percent were women
- 14 percent were ethnic minorities
- 46 percent above the age of 50
- 82 percent of full-time faculty had a master's degree; 16 percent had a doctorate
- a full-time teaching load was 15 contact hours or less per week (average is 15.3 hours) at 85 percent of two-year colleges
- 53 percent of full-time faculty participated in professional development activities offered by their college
- 38 percent of full-time faculty participated in professional development activities provided by professional associations.

Full-time faculty generally taught more higher-level courses and adjunct faculty taught more lower-level courses.

Table 2 Percent of Sections Taught by Full-Time and Adjunct Faculty in Two-Year Colleges in 2005[14]

Mathematics Course	Full-time	Adjunct
Developmental mathematics	42	56
Technical mathematics	57	37
Statistics	66	35
Precalculus	67	30
Nonmainstream calculus	75	28
Mainstream calculus for science majors	85	12
Advanced level	91	9
Service courses	71	24
Other mathematics courses	59	46

In the year 2005, adjunct faculty taught 44 percent of all two-year college mathematics sections and had these characteristics:

- 54 percent taught 6 credit hours or more
- 6 percent held doctorates and 72 percent had a master's degree as their terminal degree

- approximately 49 percent reported no employment outside the college
- approximately 25 percent taught in high school during the day
- approximately 14 percent were employed full-time in the industry.

The Standards of *Crossroads in Mathematics* (1995)

Crossroads in Mathematics outlined three sets of standards that provide the foundation for *Beyond Crossroads*:

- Standards for Intellectual Development
- Standards for Content
- Standards for Pedagogy

Standards for Intellectual Development outline guidelines for desired modes of student thinking and goals for student outcomes. All students should develop certain intellectual mathematical abilities as well as other competencies and knowledge. Introductory college mathematics courses and programs should help students see mathematics as an enriching and powerful discipline. The seven Standards for Intellectual Development outlined in *Crossroads* are presented below, with the addition of an eighth standard.

Problem solving. Students will engage in substantial mathematical problem solving.

Modeling. Students will learn mathematics through modeling real-world situations.

Reasoning. Students will expand their mathematical reasoning skills as they develop convincing mathematical arguments.

Connecting with other disciplines. Students will view mathematics as a growing discipline, interrelated with human culture, and understand its connections to other disciplines.

Communicating. Students will acquire the ability to read, write, listen to, and speak mathematics.

Using technology. Students will use appropriate technology to enhance their mathematical thinking and understanding, solve mathematical problems, and judge the reasonableness of their results.

Developing mathematical power. Students will engage in rich experiences that encourage independent, nontrivial exploration in mathematics, develop and reinforce tenacity and confidence in their abilities to use mathematics, and be inspired them to pursue the study of mathematics and related disciplines.

Linking multiple representations. Students will select, use, and translate among mathematical representations—numerical, graphical, symbolic, and verbal—to organize information and solve problems using a variety of techniques.

Standards for Content outline guidelines for selecting the content that will be taught. "Knowing mathematics" means being able to *do* mathematics. Students gain the power to solve meaningful problems through in-depth study of mathematics topics. The meaning and use of mathematical ideas should be emphasized and attention to rote manipulation deemphasized. Following are the seven Standards for Content outlined in *Crossroads* with some revision.

Number sense. Students will perform arithmetic operations, as well as reason and draw conclusions from numerical information.

Symbolism and algebra. Students will understand the use of algebraic symbolism, be able to translate problem situations into symbolic representations, and use those representations to solve problems.

Geometry and measurement. Students will develop a spatial and measurement sense, learn to visualize and use geometric models, recognize measurable attributes, and use and convert units of measure.

Function sense. Students will demonstrate understanding of the concept of function—numerically, graphically, symbolically, and verbally—and incorporate this concept into their use of mathematics.

Continuous and discrete models. Students will be able to recognize and use continuous and discrete models to solve real-world problems.

Data analysis, statistics, and probability. Students will collect, organize, analyze, and interpret data, and use that information to make informed decisions.

Deductive proof. Students will appreciate the deductive nature of mathematics as an identifying characteristic of the discipline, recognize the roles of definitions, axioms, and theorems, and identify and construct valid deductive arguments.

Standards for Pedagogy outline guidelines for instructional strategies in active student learning. Instructional strategies have a dramatic impact on what students learn. Students should understand mathematics as opposed to performing memorized procedures. Knowledge cannot be "given" to students. Students should construct their own knowledge, and monitor and guide their own learning and thinking. The five Standards for Pedagogy outlined in *Crossroads* are presented below, with some revision.

Teaching with technology. Mathematics faculty will model the use of appropriate technology in the teaching of mathematics so that students can benefit from the opportunities technology presents as a medium of instruction.

Active and interactive learning. Mathematics faculty will foster interactive learning through student writing, reading, speaking, and collaborative activities so that students can learn to work effectively in groups and communicate about mathematics both orally and in writing.

Making connections. Mathematics faculty will actively involve students in meaningful mathematics problems that build upon their experiences, focus on broad mathematical themes, and build connections within branches of mathematics and between mathematics and other disciplines.

Using multiple strategies. Mathematics faculty will use multiple instructional strategies, such as interactive lecturing, presentations, guided discovery, teaching through questioning, and collaborative learning to help students learn mathematics.

Experiencing mathematics. Mathematics faculty will provide learning activities, including projects and apprenticeships, that promote independent thinking and require sustained effort.

Conclusion

The teaching and learning of mathematics in lower division mathematics courses and assessing what is successful learning presents ongoing challenges to students, faculty, departments, and institutions. Professionals continue to search for strategies to address a variety of issues facing mathematics education such as the following:

- access, equity, and the needs of underrepresented groups
- quantitative literacy
- the choice of appropriate mathematics content and effective instructional strategies
- assessment of student learning
- the use of technology in instruction

- distance learning
- the connection between mathematics content and other disciplines, business, and industry
- negative attitudes, perceptions, and anxiety towards mathematics
- teacher preparation
- students enrolled simultaneously in high school and college (dual enrollment)
- classroom research
- professional development for full-time and adjunct faculty and instructional support staff
- collaboration with other colleges, universities, business and industry, and the public.

The environment for learning and teaching mathematics in higher education continues to change. Mathematics in the first two years of college holds the promise of opening paths to mathematical power and adventure for a segment of the student population whose opportunities might otherwise be limited. Mathematics education at this level plays such a critical role in fulfilling people's careers in a global, technological society, that its improvement is essential not only to each individual, but also to the nation. *Beyond Crossroads* challenges all faculty, departments, and institutions to adopt a philosophy that includes informed decision making and continuous improvement in order to implement the principles and standards presented in the following chapters.

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1 American Mathematical Association of Two-Year Colleges (AMATYC). Cohen, D. (Ed.). (1995). *Crossroads in Mathematics: Standards for Introductory College Mathematics Before Calculus*. Memphis, TN: American Mathematical Association of Two-Year Colleges.

2 Tate, W. F. (September 2003). SIRG Brief 2: *What Is a Standard?* National Council of Teachers of Mathematics Research Catalyst Conference, p. 21. Retrieved 3/31/2006 from www.nctm.org/highered/sirg/sirg2.pdf.

3 Weiss, I., Knapp, M. S., Hollweg, K. S., & Burrill, G. (Eds). (2002). *Investigating the Influence of Standards: A Framework for Research in Mathematics, Science and Technology*. Washington, DC: National Academy Press, p. 4.

4 Tate (2003). *Delivery standards* are human and material resources required to ensure that all students have an opportunity to learn and excel. *Resource standards* are those resources that are directly necessary for implementing an overall change strategy. *Performance standards* operationalize content standards by indicating specifically what students must do to demonstrate that they have achieved the standard.

5 Fuhrman, S. (2001). Introduction. In S. H. Fuhrman (Ed.), *From the Capital to the Classroom: Standards-based Reform in the States*. One Hundredth Yearbook of the National Society for the Study of Education, Part II. Chicago, IL: University of Chicago Press.

6 Smith, M. S. & O'Day, J. (1991). Systemic School Reform. In S. H. Fuhrman & B. Malen (Eds.), *The Politics of Curriculum and Testing*. Bristol, PA: Falmer, pp. 233–267.

7 National Council of Teachers of Mathematics (NCTM). (2000). *Principles and Standards for School Mathematics*. Reston, VA: National Council of Teachers of Mathematics.

8 Mathematical Association of America (MAA). (2004). *Undergraduate Programs and Courses in the Mathematics Sciences: CUPM Curriculum Guide 2004*. A report of the Committee on the Undergraduate Program. Washington, DC: Mathematical Association of America.

9 Joliet Junior College (IL) celebrated its 100th anniversary in 2001.

10 Phillippe, K. A. & Sullivan, L. G. (2005). *National Profile of Community Colleges: Trends and Statistics, Fourth Edition*. Washington, DC: Community College Press, American Association of Community Colleges, p. 12.

11 Ibid, pp. 12, 39, 51, 53. Statistics for numbers of students employed and working full-time are from the 2003–2004 academic year.

12 Kirkman, E., Lutzer, D. J., Maxwell, J. W., & Rodi, S. B. (to appear 2007). *Statistical Abstract of Undergraduate Programs in the Mathematical Sciences in the United States, Fall 2005 CBMS Survey*. Providence, RI: The American Mathematical Society. The final publication of *CBMS2005*, the ninth CBMS survey conducted in the fall term of 2005, will appear in print and in pdf form in 2007. The authors provided the CBMS2005 data for inclusion in *Beyond Crossroads*. CBMS2000 and CBMS2005 (when available) can be accessed for download at http://www.ams.org/cbms/.

13 Ibid.

14 Ibid.