Beyond Crossroads

Implementing Mathematics Standards in the First Two Years of College

Prepared by the *Beyond Crossroads* Writing Team
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National Association for Developmental Education

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Endorsements

The following professional organizations and AMATYC affiliates reviewed *Beyond Crossroads: Implementing Mathematics Standards in the First Two Years of College* and endorse the spirit and philosophy of the document.

American Association of Community Colleges

American Mathematical Association of Two-Year Colleges

American Statistical Association/American Mathematical Association of Two-Year Colleges Joint Committee

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National Council of Teachers of Mathematics

Ohio Mathematics and Science Coalition

AMATYC Affiliates:

Alaska Mathematical Association of Two-Year Colleges

Arizona Mathematical Association of Two-Year Colleges

California Mathematics Council, Community Colleges

California Mathematics Council, Community Colleges–South

Colorado Mathematical Association of Two-Year Colleges

Delaware Mathematical Association of Two-Year Colleges

Florida Two-Year Colleges Mathematics Association

Georgia Mathematical Association of Two-Year Colleges

Illinois Mathematics Association of Two-Year Colleges

Indiana Regional Mathematics Consortium
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New Mexico Mathematical Association of Two-Year Colleges
New York State Mathematics Association of Two-Year Colleges
North Carolina Mathematical Association of Two-Year Colleges
Ohio Mathematical Association of Two-Year Colleges
Oklahoma College Mathematical Association
Ontario College Mathematics Association
Oregon Mathematical Association of Two-Year Colleges
Minnesota Mathematical Association of Two-Year Colleges
Pacific Islands Mathematical Association of Two-Year Colleges
Pennsylvania State Mathematics Association of Two-Year Colleges
South Carolina Mathematical Association of Two-Year Colleges
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Texas Mathematical Association of Two-Year Colleges
Virginia Mathematical Association of Two-Year Colleges
Washington Mathematics Association of Two-Year Colleges
West Virginia Mathematical Association of Two-Year Colleges
Wisconsin Mathematical Association of Two-Year Colleges
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Preface

The seeds of *Beyond Crossroads: Implementing Mathematics Standards in the First Two Years of College* were planted in 2001 when a plan was drafted to revisit AMATYC's 1995 publication, *Crossroads in Mathematics: Standards for Introductory College Mathematics*. The project's National Advisory Committee, with representation from major mathematics and mathematics education organizations, met in June 2002 to advise the project leadership on the vision, direction, and general shape of the document. Surveys of members about how they used the 1995 *Crossroads* document and their needs resoundingly spoke to the importance of addressing implementation in the update of the AMATYC standards.

The AMATYC membership played a key role in helping to create and respond to various drafts. This document reflects the valuable contributions of hundreds. Through numerous forums and working sessions at annual conferences, focus sessions at Affiliate Conferences, and online questionnaires, the entire AMATYC membership and Association Review Groups and other professional organizations had many opportunities to give feedback on *Beyond Crossroads*. This is the strength of the document: that it was written by AMATYC and speaks for AMATYC and our profession, along with our professional colleagues, in bold and visionary ways.

Electronic Resources

Accompanying this document are electronic resources in various formats providing successful classroom models, instructional strategies, and other materials, intended to provide direction to faculty, departments, and institutions that wish to engage their students in rich and purposeful mathematics. These resources enhance and extend the messages of this written document, promote the implementation of the standards, and provide a "living" link between this document and its readers through providing current information on initiatives and resources supporting the Implementation Standards and recommendations. The initial set of resources includes a Web-enhanced version of this document, *Beyond Crossroads Live!*, an Outreach Kit, and resources on Assessment and Quantitative Literacy/Mathematics Across the Curriculum. For more information about *Beyond Crossroads* electronic resources, visit the AMATYC Web site, www.amatyc.org. Electronic files of the *Beyond Crossroads* written document are also available at the AMATYC Web site, www.amatyc.org.

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Chapter 1

Beyond Crossroads: Implementing Mathematics Standards in the First Two Years of College

Shaping the Vision

In 1995, the American Mathematical Association of Two-Year Colleges (AMATYC) released its first standards document, *Crossroads in Mathematics*.[1] *Crossroads in Mathematics* emphasized desired modes of student thinking and guidelines for selecting content and instructional strategies. The purposes of AMATYC's second standards document, *Beyond Crossroads*, are to renew and to extend the goals, principles, and standards set forth in *Crossroads* and to continue the call for their implementation. *Beyond Crossroads* presents a renewed vision for mathematics courses offered in the first two years of college with an additional set of standards, called Implementation Standards, which focus on student learning and the learning environment, assessment of student learning, curriculum and program development, instruction, and professionalism.

Beyond Crossroads is intended to stimulate faculty, departments, and institutions to examine, assess, and improve every component of mathematics education in the first two years of college. The varied challenges for full-time and adjunct faculty are fully acknowledged. Faculty need and deserve the necessary facilities, equipment, and professional development opportunities essential for performing their teaching responsibilities. These standards, recommendations, and action items are not intended to be a prescription for action used identically by each faculty member, department, or institution. Rather, they are to be used as a starting point for dialogue, reflection, experimentation, evaluation, and continuous improvement. Used in this way, this document can guide professionals toward standards-based mathematics education that promotes continuous professional growth and helps students maximize their potential in every college mathematics course.

> **The ultimate goals of this document are to improve mathematics education and to encourage more students to study mathematics.**
>
> *Crossroads in Mathematics: Standards for Introductory College Mathematics Before Calculus*, AMATYC, 1995, p. 1

What is a mathematics standard? Mathematics faculty, administrators, mathematics education researchers, policy makers, politicians, and parents continue to engage in dialogue on the meaning and role of standards. The words "educational standard" can have any of the following meanings depending on the audience and the purpose for which the standard is developed:[2]

- a vision of ideal practice
- essential knowledge in a field
- descriptors of student performance
- guides to align system components
- measurable goals for student learning

- clear curricular goals for instructors
- guides in measurement and accountability systems
- mechanisms that encourage dialogue and consensus building.

"Nationally developed standards in mathematics, science, and technology represent a set of fundamental changes in the way these subjects have traditionally been taught..."[3] The Standards for Intellectual Development, Content, and Pedagogy of *Crossroads in Mathematics* focused on content and instruction, describing what students should know and be able to do in mathematics and outlining pedagogical principles. To accomplish those standards, additional levels of support are needed.[4] The Implementation Standards of *Beyond Crossroads* address the key elements of student expectations, facilities, human resources, materials, curriculum design, instructional delivery formats, and professional development to achieve the standards of *Crossroads in Mathematics*.

One of the goals of *Beyond Crossroads* is to clarify issues, interpret, and translate research to bring standards-based mathematics instruction into practice. Two definitions from the literature of standards-based education have been adopted:

- Standards-based education entails implementing strategies (related to learning, assessment, curricula, teaching, and professionalism) and policies of what students should know and be able to do.[5]
- Standards-based education implies a greater coherence, or alignment, among the parts of the educational systems.[6] The assumption is that components/strategies that are aligned are more likely to be successful.

To accomplish this alignment, *Beyond Crossroads* has integrated recommendations from AMATYC position statements and related mathematics organizations: *Principles and Standards for School Mathematics* of the National Council of Teachers of Mathematics (NCTM)[7] and *Undergraduate Programs and Courses in the Mathematics Sciences: CUPM Curriculum Guide 2004*, a report of the Committee on the Undergraduate Program in Mathematics of the Mathematical Association of America (MAA).[8]

The Implementation Standards of *Beyond Crossroads* are first presented in Chapter 3. Then, each of Chapters 4 through 8 focuses on one Implementation Standard. Sections within those chapters address key issues for implementing standards-based mathematics education and include the following:

- supporting research on key issues
- lists of expectations of students to be communicated by faculty to help students set ambitious goals for themselves, accept responsibility for their own learning, and achieve greater success in their mathematics courses
- implementation recommendations
- action items for faculty offer guidance for designing curricula, choosing instructional strategies, and continuing to develop and contribute to the profession throughout their careers
- action items for departments and institutions outline ways to improve the learning environment, develop curricula, and support faculty.

The focus of the document is two-year college mathematics education and lower division mathematics education at four-year colleges and universities with characteristics similar to two-year colleges. The primary audience for *Beyond Crossroads* is two-year college mathematics faculty.[i] Since faculty who teach lower division mathematics at four-year colleges and universities are faced with similar issues as two-year college faculty, they are also an important audience. Additional audiences include college administrators, K–12 teachers, policy makers, government agencies, professional societies, publishers,

[i] Throughout this document, the term *faculty* will refer to both full-time and adjunct (part-time) professional teaching staff.

and funding agencies. Individuals, organizations, and businesses outside of education are called upon in *Beyond Crossroads* to collaborate with educators and each other, to improve college mathematics programs, and to respond to the needs of the mathematics community. This document serves as a call to action for all stakeholders to work together to improve student success in mathematics courses and programs in the first two years of college, with particular emphasis on the two-year college student, faculty, and institution characteristics.

> **I advise my students to listen carefully the moment they decide to take no more mathematics courses. They might be able to hear the sound of closing doors.**
>
> James Caballero, Everybody a Mathematician? CAIP Quarterly, Fall 1989, 2(2), p. 2

Distinctive Characteristics of Two-Year Colleges, Students, and Faculty

The distinctive characteristics of two-year colleges, students, and faculty make a compelling case for the development and implementation of distinguishing standards for mathematics in the first two years of college. In the century since its inception,[9] the two-year college has grown to offer a wide range of transfer, technical, and career-specific courses and programs to a diverse student population.

There were more than 1,150 two-year colleges serving 10.1 million students, with 6.6 million enrolled in credit classes in the year 2005. A two-year college was within commuting distance of nearly every person in the United States. In the academic year 2001–2002, 53 percent of all undergraduate students in the United States were enrolled at two-year colleges.[10] Most two-year colleges offered the following courses, programs, and services:

- open-door admission
- extensive developmental education programs
- a wide range of lower-division undergraduate courses that transfer to four-year colleges
- two-year associate degree programs in the arts and the sciences
- two-year associate degree programs in technical careers designed to meet critical local economic needs
- certificate programs for training in entry-level job skills
- short-term job training and continuing education related to local employment and economic growth
- adult literacy education and high school GED diplomas
- mandatory placement testing in mathematics for 98% of first-time mathematics students with mandatory placement at two-thirds of the colleges
- mathematics tutoring labs, staffed by full-time or adjunct faculty, paraprofessionals, or peer tutors.

In the academic year 2001–2002, two-year college students had these characteristics:[11]

- the average age was 29; 36 percent were 18–21 years old; 15 percent were 40 years or older
- 58 percent were women and thirty-three percent were minority students (black, native American, Asian/Pacific Islander, Hispanic)
- 61 percent of all students took a part-time course load
- 80 percent were employed with 41 percent employed full-time
- many two-year college students were involved in a career change, had not attended school in several years, and were commuters.

Approximately 1.3 million students enrolled in the following mathematics courses at two-year colleges in fall 2005.

Table 1 Percent of Students Enrolled in Mathematics Courses at Two-Year Colleges in 2005[12]

Mathematics Course	Percent
Developmental mathematics (precollege)	57
Precalculus	19
Calculus	6
Statistics	7
Other mathematics courses*	11

*These include linear algebra, probability, discrete mathematics, finite mathematics, mathematics for liberal arts, mathematics for elementary school teachers, technical mathematics, and computing.

The 8,793 full-time permanent faculty teaching mathematics in two-year colleges in the year 2005 had the following characteristics:[13]

- 44 percent were women
- 14 percent were ethnic minorities
- 46 percent above the age of 50
- 82 percent of full-time faculty had a master's degree; 16 percent had a doctorate
- a full-time teaching load was 15 contact hours or less per week (average is 15.3 hours) at 85 percent of two-year colleges
- 53 percent of full-time faculty participated in professional development activities offered by their college
- 38 percent of full-time faculty participated in professional development activities provided by professional associations.

Full-time faculty generally taught more higher-level courses and adjunct faculty taught more lower-level courses.

Table 2 Percent of Sections Taught by Full-Time and Adjunct Faculty in Two-Year Colleges in 2005[14]

Mathematics Course	Full-time	Adjunct
Developmental mathematics	42	56
Technical mathematics	57	37
Statistics	66	35
Precalculus	67	30
Nonmainstream calculus	75	28
Mainstream calculus for science majors	85	12
Advanced level	91	9
Service courses	71	24
Other mathematics courses	59	46

In the year 2005, adjunct faculty taught 44 percent of all two-year college mathematics sections and had these characteristics:

- 54 percent taught 6 credit hours or more
- 6 percent held doctorates and 72 percent had a master's degree as their terminal degree

- approximately 49 percent reported no employment outside the college
- approximately 25 percent taught in high school during the day
- approximately 14 percent were employed full-time in the industry.

The Standards of *Crossroads in Mathematics* (1995)

Crossroads in Mathematics outlined three sets of standards that provide the foundation for *Beyond Crossroads*:

- Standards for Intellectual Development
- Standards for Content
- Standards for Pedagogy

Standards for Intellectual Development outline guidelines for desired modes of student thinking and goals for student outcomes. All students should develop certain intellectual mathematical abilities as well as other competencies and knowledge. Introductory college mathematics courses and programs should help students see mathematics as an enriching and powerful discipline. The seven Standards for Intellectual Development outlined in *Crossroads* are presented below, with the addition of an eighth standard.

Problem solving. Students will engage in substantial mathematical problem solving.

Modeling. Students will learn mathematics through modeling real-world situations.

Reasoning. Students will expand their mathematical reasoning skills as they develop convincing mathematical arguments.

Connecting with other disciplines. Students will view mathematics as a growing discipline, interrelated with human culture, and understand its connections to other disciplines.

Communicating. Students will acquire the ability to read, write, listen to, and speak mathematics.

Using technology. Students will use appropriate technology to enhance their mathematical thinking and understanding, solve mathematical problems, and judge the reasonableness of their results.

Developing mathematical power. Students will engage in rich experiences that encourage independent, nontrivial exploration in mathematics, develop and reinforce tenacity and confidence in their abilities to use mathematics, and be inspired them to pursue the study of mathematics and related disciplines.

Linking multiple representations. Students will select, use, and translate among mathematical representations—numerical, graphical, symbolic, and verbal—to organize information and solve problems using a variety of techniques.

Standards for Content outline guidelines for selecting the content that will be taught. “Knowing mathematics” means being able to *do* mathematics. Students gain the power to solve meaningful problems through in-depth study of mathematics topics. The meaning and use of mathematical ideas should be emphasized and attention to rote manipulation deemphasized. Following are the seven Standards for Content outlined in *Crossroads* with some revision.

Number sense. Students will perform arithmetic operations, as well as reason and draw conclusions from numerical information.

Symbolism and algebra. Students will understand the use of algebraic symbolism, be able to translate problem situations into symbolic representations, and use those representations to solve problems.

Geometry and measurement. Students will develop a spatial and measurement sense, learn to visualize and use geometric models, recognize measurable attributes, and use and convert units of measure.

Function sense. Students will demonstrate understanding of the concept of function—numerically, graphically, symbolically, and verbally—and incorporate this concept into their use of mathematics.

Continuous and discrete models. Students will be able to recognize and use continuous and discrete models to solve real-world problems.

Data analysis, statistics, and probability. Students will collect, organize, analyze, and interpret data, and use that information to make informed decisions.

Deductive proof. Students will appreciate the deductive nature of mathematics as an identifying characteristic of the discipline, recognize the roles of definitions, axioms, and theorems, and identify and construct valid deductive arguments.

Standards for Pedagogy outline guidelines for instructional strategies in active student learning. Instructional strategies have a dramatic impact on what students learn. Students should understand mathematics as opposed to performing memorized procedures. Knowledge cannot be "given" to students. Students should construct their own knowledge, and monitor and guide their own learning and thinking. The five Standards for Pedagogy outlined in *Crossroads* are presented below, with some revision.

Teaching with technology. Mathematics faculty will model the use of appropriate technology in the teaching of mathematics so that students can benefit from the opportunities technology presents as a medium of instruction.

Active and interactive learning. Mathematics faculty will foster interactive learning through student writing, reading, speaking, and collaborative activities so that students can learn to work effectively in groups and communicate about mathematics both orally and in writing.

Making connections. Mathematics faculty will actively involve students in meaningful mathematics problems that build upon their experiences, focus on broad mathematical themes, and build connections within branches of mathematics and between mathematics and other disciplines.

Using multiple strategies. Mathematics faculty will use multiple instructional strategies, such as interactive lecturing, presentations, guided discovery, teaching through questioning, and collaborative learning to help students learn mathematics.

Experiencing mathematics. Mathematics faculty will provide learning activities, including projects and apprenticeships, that promote independent thinking and require sustained effort.

Conclusion

The teaching and learning of mathematics in lower division mathematics courses and assessing what is successful learning presents ongoing challenges to students, faculty, departments, and institutions. Professionals continue to search for strategies to address a variety of issues facing mathematics education such as the following:

- access, equity, and the needs of underrepresented groups
- quantitative literacy
- the choice of appropriate mathematics content and effective instructional strategies
- assessment of student learning
- the use of technology in instruction

- distance learning
- the connection between mathematics content and other disciplines, business, and industry
- negative attitudes, perceptions, and anxiety towards mathematics
- teacher preparation
- students enrolled simultaneously in high school and college (dual enrollment)
- classroom research
- professional development for full-time and adjunct faculty and instructional support staff
- collaboration with other colleges, universities, business and industry, and the public.

The environment for learning and teaching mathematics in higher education continues to change. Mathematics in the first two years of college holds the promise of opening paths to mathematical power and adventure for a segment of the student population whose opportunities might otherwise be limited. Mathematics education at this level plays such a critical role in fulfilling people's careers in a global, technological society, that its improvement is essential not only to each individual, but also to the nation. *Beyond Crossroads* challenges all faculty, departments, and institutions to adopt a philosophy that includes informed decision making and continuous improvement in order to implement the principles and standards presented in the following chapters.

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1 American Mathematical Association of Two-Year Colleges (AMATYC). Cohen, D. (Ed.). (1995). *Crossroads in Mathematics: Standards for Introductory College Mathematics Before Calculus*. Memphis, TN: American Mathematical Association of Two-Year Colleges.

2 Tate, W. F. (September 2003). SIRG Brief 2: *What Is a Standard?* National Council of Teachers of Mathematics Research Catalyst Conference, p. 21. Retrieved 3/31/2006 from www.nctm.org/highered/sirg/sirg2.pdf.

3 Weiss, I., Knapp, M. S., Hollweg, K. S., & Burrill, G. (Eds). (2002). *Investigating the Influence of Standards: A Framework for Research in Mathematics, Science and Technology*. Washington, DC: National Academy Press, p. 4.

4 Tate (2003). *Delivery standards* are human and material resources required to ensure that all students have an opportunity to learn and excel. *Resource standards* are those resources that are directly necessary for implementing an overall change strategy. *Performance standards* operationalize content standards by indicating specifically what students must do to demonstrate that they have achieved the standard.

5 Fuhrman, S. (2001). Introduction. In S. H. Fuhrman (Ed.), *From the Capital to the Classroom: Standards-based Reform in the States*. One Hundredth Yearbook of the National Society for the Study of Education, Part II. Chicago, IL: University of Chicago Press.

6 Smith, M. S. & O'Day, J. (1991). Systemic School Reform. In S. H. Fuhrman & B. Malen (Eds.), *The Politics of Curriculum and Testing*. Bristol, PA: Falmer, pp. 233–267.

7 National Council of Teachers of Mathematics (NCTM). (2000). *Principles and Standards for School Mathematics*. Reston, VA: National Council of Teachers of Mathematics.

8 Mathematical Association of America (MAA). (2004). *Undergraduate Programs and Courses in the Mathematics Sciences: CUPM Curriculum Guide 2004*. A report of the Committee on the Undergraduate Program. Washington, DC: Mathematical Association of America.

9 Joliet Junior College (IL) celebrated its 100th anniversary in 2001.

10 Phillippe, K. A. & Sullivan, L. G. (2005). *National Profile of Community Colleges: Trends and Statistics, Fourth Edition*. Washington, DC: Community College Press, American Association of Community Colleges, p. 12.

11 Ibid, pp. 12, 39, 51, 53. Statistics for numbers of students employed and working full-time are from the 2003–2004 academic year.

12 Kirkman, E., Lutzer, D. J., Maxwell, J. W., & Rodi, S. B. (to appear 2007). *Statistical Abstract of Undergraduate Programs in the Mathematical Sciences in the United States, Fall 2005 CBMS Survey*. Providence, RI: The American Mathematical Society. The final publication of *CBMS2005*, the ninth CBMS survey conducted in the fall term of 2005, will appear in print and in pdf form in 2007. The authors provided the CBMS2005 data for inclusion in *Beyond Crossroads*. CBMS2000 and CBMS2005 (when available) can be accessed for download at http://www.ams.org/cbms/.

13 Ibid.

14 Ibid.

Chapter 2

Guiding Principles

Embracing Change

Implementing the standards of *Beyond Crossroads* includes making a commitment to continuous improvement and professional development. It may also involve changing actions and philosophies of faculty, departments, and institutions. These changes may involve a relatively small, piecemeal change, or may be a large-scale, systemic change. "Systemic change is a cyclical process in which the impact of change on all parts of the whole and their relationship to one another are taken into consideration...it is not so much a detailed prescription for improving education as a philosophy advocating reflecting, rethinking, and restructuring."[1] Systemic change may be incremental (small operational changes), substantial (key functions are overhauled), or transformational (changing the entity irreversibly).[2]

Indeed, the challenges of making and embracing change are numerous. *Beyond Crossroads* advocates developing goals and objectives based on input from all stakeholders, so that all students can be successful learners of ambitious learning outcomes in mathematics. A small, piecemeal change might be appropriate in a particular activity. In another situation, large-scale systemic change might be necessary to completely define new goals, objectives, and strategies, and essentially start over.

> **Educational excellence has become a moving target. While basic skills such as reading, writing and math will likely remain at the core of the curriculum, the abilities built on this foundation continue to change with our society.**
>
> National School Boards Foundation, *Education Leadership Toolkit: Change and Technology in America's Schools,* 2005, p. 1.

For some professionals, embracing and implementing change is natural or even intuitive. They look for ways to improve and revise instructional strategies and curricula on a regular basis. For them, this process is comfortable. For others, *any* change can be challenging. They feel most comfortable and confident when they can predict what will happen in their classroom and are familiar with student responses and difficulties. Regardless of whether the process of change is natural or challenging, continuous improvement in mathematics instruction is essential to improving student learning.

Embracing the challenge of continuously improving student learning is a component of professional growth. To continue to grow professionally, faculty need to look inward and engage in a process that assesses their own teaching practice regularly. Then they need to use that information to redesign goals, objectives, and student learning activities. Implementing standards may involve stepping outside of one's comfort zone, experimenting with new methods, and building confidence. Embracing change and continuous improvement are reflective processes of planning, implementing, evaluating, and documenting, followed by redefining, implementing again, and refining the action in the future to improve student learning in mathematics.

Basic Principles of *Beyond Crossroads*

Addressing the issues and challenges facing mathematics education in the first two years of college begins by adopting a set of basic principles. These principles are the foundation upon which all of the *Crossroads* standards are built. The Basic Principles outlined in *Crossroads in Mathematics* in 1995 have been revisited, updated, and expanded to form the philosophical underpinnings of *Beyond Crossroads* and are presented here in alphabetical order.

- ***Assessment.*** *The assessment of student learning in mathematics should be a fundamental tool for the improvement of instruction and student learning*.

  Assessment should support mathematics learning and instruction. An effective assessment program includes assessment of learning outcomes at the class, course, and program levels of instruction.

- ***Broadening.*** *Mathematics courses and programs in the first two years of college should broaden students' options in educational and career choices*.

  The mathematical content, reasoning skills, and communication skills developed in mathematics courses should open doors for students to pursue future work in a variety of fields.

- ***Equity and access.*** *All students should have equitable access to high-quality, challenging, effective mathematics instruction and support services*.

  The mathematics education community must reach out to all students. Active participation of all students in mathematics and the pursuit of mathematics-intensive careers by many are critical goals of our society.

- ***Innovation.*** *Mathematics programs should be thoughtfully constructed to approach content and instruction with appropriate use of traditional and innovative methods*.

  Mathematics content and instruction should include opportunities for students to engage in inquiry, problem solving, modeling, and collaborative learning, using appropriate technology. Thoughtfully crafted interactive lectures can highlight meaningful mathematics and inspire students to learn.

- ***Inquiry.*** *Effective mathematics instruction should require students to be active participants*.

  Students learn through investigation. Advances in neuroscience confirm that students' active involvement in learning mathematics is important in the process of building understanding and modifying the structure of the mind.[3]

- ***Quantitative literacy.*** *Quantitative literacy should be integrated throughout the mathematics program and the college curricula*.

  Quantitative literacy is "the capacity to identify, understand and engage in mathematics as well as make well-founded mathematical judgments about the role that mathematics plays in individual's current and future life as a constructive, concerned and reflective citizen."[4] Students' insight and skills for solving quantitative problems in context should be developed throughout the entire college curricula.

- ***Relevance.*** *The mathematics that students study should be meaningful and foster their appreciation of the discipline*.

  Mathematics should be presented in the context of realistic, understandable, applied problems that help students develop an appreciation of the nature, history, and usefulness of the discipline.

- ***Research into practice.*** *The practice of mathematics teaching should be guided by research on teaching and learning*.

  Faculty are best prepared to design effective mathematics instructional strategies and assessment tools when they have an understanding of the results of pertinent educational research, particularly in cognitive science and learning theory, and when they use those results to make informed decisions about their teaching.

✦ ***Technology.*** *Technology should be integral to the teaching and learning of mathematics*.

Technology continues to change the face of mathematics and affect the relative importance of various concepts and topics of the discipline. Advancements in technology have changed not only *how* faculty teach, but also *what* is taught and *when* it is taught. Using some of the many types of technologies can deepen students' learning of mathematics and prepare them for the workplace.

Conclusion

To address the challenges facing mathematics education in the first two years of college, *Beyond Crossroads* focuses on implementation of the *Crossroads in Mathematics* Standards and advocates for informed decision-making. Faculty need to embrace change and use a multifaceted approach to instruction, not only to address a variety of student learning styles, but also to reveal the richness and interconnectedness of mathematics. *Beyond Crossroads* introduces five Implementation Standards in the next chapter, extending the Principles and Standards for Intellectual Development, Content, and Pedagogy presented in *Crossroads in Mathematics*. These standards, collectively, call for all faculty to challenge themselves professionally and to empower students quantitatively to be successful in their lives and careers.

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1 National School Boards Foundation. (2005). *Education Leadership Toolkit: Change and Technology in America's Schools, What Is Systemic Change?* Retrieved 3/30/2006 from http://www.nsba.org/sbot/toolkit/whatsc.html, p. 1.

2 Foster, R. N. & Kaplan, S. (2001). *Creative Destruction*. McKinsey & Company: Currency Publishing.

3 National Research Council. (1999). Bransford, J. D., Brown, A. L. & Cocking, R. R. (Eds.). *How People Learn: Brain, Mind, Experience, and School*. Washington, DC: National Research Council. National Academy Press, pp. 102–115.

4 Organization for Economic Cooperation and Development, UNESCO Institute for Statistics (2003). *Literacy Skills for the World of Tomorrow—Further Results from PISA 2000*. Paris, France: Organization for Economic Cooperation and Development Publishing, p. 20.

Chapter 3

The Implementation Standards of *Beyond Crossroads*

Beyond Crossroads extends the three sets of standards from *Crossroads in Mathematics* with an additional set of standards called Implementation Standards. The content of each of the following chapters focuses on one implementation standard as outlined:

- Student Learning and the Learning Environment — Chapter 4
- Assessment of Student Learning — Chapter 5
- Curriculum and Program Development — Chapter 6
- Instruction — Chapter 7
- Professionalism — Chapter 8

Each section within a chapter presents research and rationale, expectations of students, recommendations, and action items for faculty, departments, and institutions for a particular issue in mathematics education in the first two years of college. The following Implementation Standards are guidelines for faculty, departments, and institutions for improving mathematics education in the first two years of college for all students—those underprepared in mathematics, as well as those ready to enter, or bypass, beginning calculus.

Implementation Standard: Student Learning and the Learning Environment

Mathematics faculty and their institutions will create an environment that optimizes the learning of mathematics for all students.

Two-year colleges serve a student body with varied characteristics and academic needs. Each student is entitled to the best educational experiences and opportunities available regardless of age, gender, sexual orientation, race and cultural differences, socio-economic factors, physical and cognitive abilities, or pre-college experiences. Creating a learning environment that maximizes student learning in mathematics and responds to the needs of all students requires the active involvement of every faculty member and each component of the institution.

Implementation Standard: Assessment of Student Learning

Mathematics faculty will use the results from the ongoing assessment of student learning of mathematics to improve curricula, materials, and teaching methods.

Assessment of student learning of mathematics should be aligned with curriculum and instruction to support student learning. Effective assessment practices include the documentation of student learning at the class, course, and program level.

Implementation Standard: Curriculum and Program Development

Mathematics departments will develop, implement, evaluate, assess, and revise courses, course sequences, and programs to help students attain a higher level of quantitative literacy and achieve their academic and career goals.

Mathematics departments, in collaboration with appropriate stakeholders, should regularly engage in course and program review and evaluation to assure that the mathematics curricula prepare students to be numerate citizens and productive employees who have an appreciation for mathematics and lifelong learning. These course and program reviews and the subsequent revisions should reflect the department's own analysis of student achievement and the informed practice of the mathematics community.

Implementation Standard: Instruction

Mathematics faculty will use a variety of instructional strategies that reflect the results of research to enhance student learning.

Effective mathematics instruction requires a variety of resources, materials, technology, and delivery systems that take into account students' different learning styles and instructors' different teaching styles. Using multiple strategies in the classroom will increase the level of engagement of students and open opportunities for more students to be actively involved in the learning of mathematics.

Implementation Standard: Professionalism

Institutions will hire qualified mathematics faculty, and these faculty will engage in ongoing professional development and service.

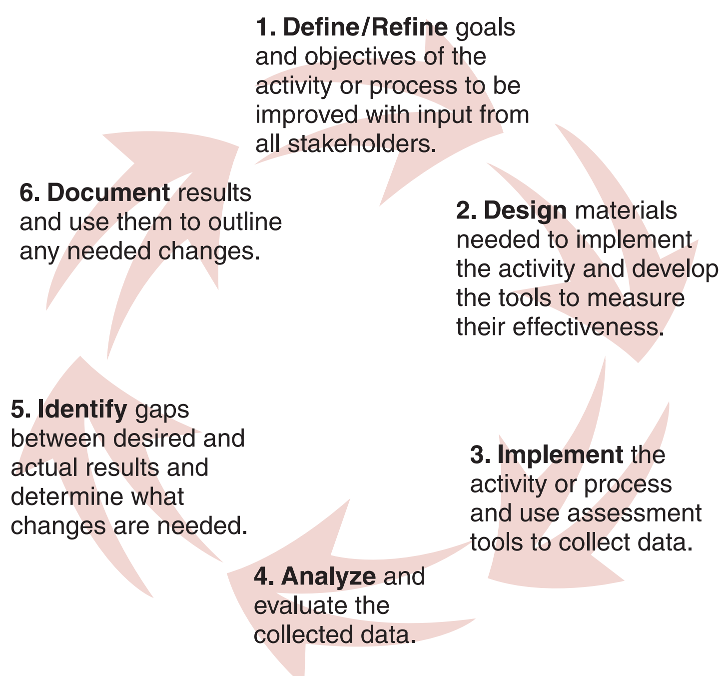
Institutions should be proactive in recruiting candidates with diverse backgrounds and hiring qualified mathematics faculty. These faculty need to continually expand their mathematics knowledge, stay current with new research on learning and teaching, and be active in the college and the profession. The institution should support mathematics faculty by providing opportunities for faculty to learn and grow in their profession.

The Implementation Cycle of *Beyond Crossroads*

As a complement to the Implementation Standards, the following six-step Implementation Cycle is presented as a model for making change and improvements in a component of mathematics education. Mathematics professionals who find implementing standards somewhat overwhelming may find that the cycle provides a step-by-step process for continuous improvement. Embracing change with this model may be more comfortable in small, manageable steps. This cycle can be adapted to any standard, program, course, activity or process—from designing the optimal classroom, assessing student learning, designing curricula, choosing an instructional strategy, to hiring a new faculty member. Like Polya's problem-solving method and other improvement models, the steps of the model remain constant, with the specific activities within each step dictated by the particular situation being implemented, evaluated, and improved.

Inherent in the implementation cycle is assessment. Evaluative data is collected, analyzed, and used to refine goals and objectives. Then the cycle begins again. This feedback loop—implementing changes, refining goals, and beginning the cycle again—is perhaps the most important, and too often neglected, phase of the change cycle. What has been learned from implementation and assessment must be used to improve the activity the next time. The dynamic feedback loop should never end.

Figure 1 The Implementation Cycle of *Beyond Crossroads*



1. **Define** and build consensus on goals and objectives of the activity or process to be implemented. Gather input from all stakeholders and advisory groups. Be sure to consider the current situation and specific aspects of the activity or process.
2. **Design** (or obtain) the materials needed to implement the activity. Develop the assessment tools and instruments to measure the achievement of goals and objectives. Identify and obtain needed equipment. Hire and train personnel.
3. **Implement** the activity or process. Use the measurement tools to collect data.
4. **Analyze** and evaluate the collected data.
5. **Identify** gaps and determine what changes are needed. Investigate the degree to which desired goals and objectives are achieved. Determine the strengths and weaknesses of each component of the activity.
6. **Document** results and use them to outline revisions in the activity or process. Agree on changes in the activities or measurement tools to meet desired goals when the activity is offered again.

Revisit step 1. Refine goals and objectives of the activity or process. Request input from stakeholders regarding revisions in the activity and proceed through the cycle again. Implementing changes and beginning the cycle again, is perhaps the most important phase of the change cycle.

Periodic reviews and input from all stakeholders assure that curriculum and instruction are dynamic and the ever-changing needs of all students are addressed. To assess progress when using the Implementation Cycle, the following questions may be helpful.

Define	✓	Have key issues been clearly defined?
	✓	Have all stakeholders had an opportunity to provide input?
	✓	Have goals, objectives, and outcomes of the activity or process been clearly defined?
	✓	What can be learned from research about the activity?
Design	✓	What resources (equipment, personnel) are needed and are they available?
	✓	Have all the necessary materials been developed (or obtained)?
	✓	Have appropriate instruments been developed to assess the activity?
	✓	Have personnel been hired and trained adequately for this task?
Implement	✓	Was the activity implemented according to the agreed upon plan?
	✓	Was data collected to evaluate the outcomes?
Analyze	✓	Has the collected data been analyzed and evaluated?
Identify	✓	What was learned from the collected data?
	✓	What were the strengths and weaknesses of the activity?
	✓	What were the gaps between desired and actual outcomes?
	✓	What changes are necessary to improve the activity?
Document	✓	Were the results of the evaluation used to outline revisions in the activity or program?
	✓	Have the results of the evaluation been documented and communicated broadly to those involved?

The dynamic feedback loop, followed by outlining what has been learned from implementation and assessment to improve the activity the next time, is the key component to continuous improvement.[1]

Refine	✓	Which goals, objectives, and/or activities need to be revised?
	✓	What changes are necessary to close the gaps between desired and actual outcomes?
Begin the Cycle Again	✓	Have stakeholders been consulted again regarding revised goals, objectives, and activities?

Implementing the Vision

Thoughtful implementation of the principles and standards presented in *Beyond Crossroads* is an ongoing process. The cooperative engagement of the principles, the Intellectual Development, Content, and Pedagogy Standards, and the Implementation Standards is synergistic. Implementing a group of standards in concert has a greater effect on the improvement of student learning and the professionalism of faculty than any single standard implemented in isolation. *Beyond Crossroads* presents research about the learning and teaching of mathematics and guidance on how to implement new strategies and argues for an action-oriented vision. The standards-based mathematics education addressed in this document is intended to guide professionals in their decision-making and provide a forum for dialogue on how to design strategies to meet the challenges of improving student learning in mathematics. The goal is to empower faculty to strengthen the learning and teaching of mathematics by creating changes based on informed decision-making and goals and objectives defined with input from all stakeholders. The following chapters address that implementation process.

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[1] Other examples of continuous improvement models: The Shewhart cycle (PDCA—Plan, Do, Check, Act); The Deming cycle (PDSA—Plan, Do, Study, Act); The Six Sigma model (DMAIC—Define, Measure, Analyze, Improve, Control).

Beyond CROSSROADS

Chapter 4

Student Learning and the Learning Environment

Many factors influence the academic progress of students. The varied academic preparation of the diverse two-year college student population calls for the entire college community to work in concert to create a positive learning environment that will maximize student learning in mathematics, both inside and outside of the classroom. In a standards-based learning environment, students are viewed as partners in the learning experience. To nurture that partnership, faculty may need to help students identify their academic strengths and weaknesses, develop strategies to minimize mathematics anxiety, and learn how to take responsibility for their own learning. Faculty, departments, and institutions need to make available to students an effective mathematics placement program. Once a student has enrolled in an appropriate mathematics course, it is essential to provide a supportive learning environment consisting of the classroom, mathematics tutoring labs, learning centers, counselors, support for students with disabilities, and other support services.

Implementation Standard: Student Learning and the Learning Environment

Mathematics faculty and their institutions will create an environment that optimizes the learning of mathematics for all students.

Responding to the Needs of a Diverse Student Population

The diverse student population of two-year colleges discussed in Chapter 1 presents unique challenges for students, faculty, and institutions. Large numbers of mathematics students need institutional support such as tutoring, financial aid, and childcare. Many students have full- or part-time work responsibilities, as well as significant family and personal commitments. Students with disabilities often choose two-year colleges for their educational needs. Variations in students' mathematics achievement have long been associated with the demographic issues of socioeconomic status and race/ethnicity. First-generation, immigrant, and international students may face difficulty adjusting to a new educational environment. To address these challenges, students need to assume an active role in their learning. Faculty and institutions need to create a responsive mathematics learning environment that responds to the needs and characteristics of its students.

> **Socioeconomic status is the greatest determinant of enrollment and persistence in college for all students.**
>
> Heather Oesterreich, *Characteristics of Effective Urban College Preparation Programs,* 2000, p. 4

Each student is entitled to access and support for high-quality educational experiences and opportunities—regardless of age, gender, sexual orientation, race, ethnicity, socioeconomic factors, physical and cognitive abilities, or precollege experiences. Creating an appropriate and responsive learning environment for academic success in mathematics is a responsibility that needs to be shared by each person in the institution. The institution needs to provide leadership training and equity training, so that faculty and student support staff acquire the necessary skills to address students' needs.

Students will be expected to do the following:

- begin taking their mathematics courses early in their program
- set high mathematical expectations for themselves
- learn to balance work commitments and other responsibilities with the time needed to achieve course objectives
- recognize their physical and cognitive capabilities and apply strategies that maximize those capabilities, such as attending class regularly and completing homework assignments
- conscientiously persist in each mathematics course
- make use of all resources provided by the institution
- acquire the necessary communication skills to accomplish course goals.

Implementation recommendation: The institution will provide all students with quality educational experiences and mathematical opportunities, regardless of distinguishing characteristics or precollege experiences.

Actions to support this recommendation

Faculty actions:

- have high expectations of all students and clearly communicate those expectations to students
- be aware of and accommodate diverse student needs
- collaborate with appropriate support services personnel to respond to the needs of students with disabilities
- serve as student mentors and mathematics advisors
- advise students of the availability and appropriate use of academic support resources.

Departmental/institutional actions:

- provide mentors for students
- provide mathematics tutoring centers, tutor training, and academic counselors
- assure that students have access to needed technology, such as computer software and hardware, tape recorders, calculators, and videos
- provide appropriate support services for disabled students
- establish support structures for students who lack academic prerequisites.

Initial Placement into the Mathematics Curriculum

Appropriate placement into a mathematics course is critical to a student's success in mathematics. The outcome of that placement process influences not only the time to program completion but also the student's self-perception as a learner of mathematics. To assure the best student placement, an institutional placement policy should consider multiple factors. While testing provides important information for placement into mathematics courses, a mandatory placement policy based only on standardized test scores may effectively deny access to necessary courses for some students.[1] Measures such as placement testing, high school preparation, prior exposure to course content, time since high school graduation, enrollment status, enrollment in a study skills course, mathematical self-concept, attitude toward mathematics, attendance, and educational goals have been shown to be pertinent to student success.[2] Mathematics placement policies should place students into the highest-level course in which they are likely to be successful. An appeal process should be in place to accommodate the possibility of inaccurate placement.

Mathematics faculty involvement is essential when developing institutional placement policies prescribing initial placement of students into mathematics courses. The majority of two-year colleges in the United States rely on standardized tests for placement into the mathematics curriculum,[3] but the validity of these tests as predictors of academic success varies.[4] Initial advising on placement into the mathematics curriculum, mathematics course selection, and the implications of these choices on degree attainment are important considerations for student success and for encouraging more students to enroll in mathematics courses and programs. An advisor can help a student construct a personal profile detailing what the learner knows, wants to know, and needs to know to ensure that the appropriate mathematics class or classes are selected. Students should be encouraged to practice their mathematics skills before taking a placement test, in order to review previously learned concepts. Conversations between students and advisors about course placement engage students as full partners in their education.

If the student is not initially placed into a mathematics class appropriately, then the student is at an immediate disadvantage before other factors have the opportunity to influence success.

Phyllis S. Shaw, *An Analysis and Evaluation of Policies and Practices of Student Placement into College Algebra Classes...*, Doctoral Dissertation, 1997, p. 6

Students will be expected to do the following:

- prepare to take a placement test by reviewing test preparation materials supplied by the college
- be aware of the implications of placement results.

Implementation recommendation: A college placement team that includes mathematics faculty will develop policies and procedures for the placement of students into the mathematics curriculum based on an analysis of multiple measures.

Actions to support this recommendation

Faculty actions:

- be familiar with mathematics placement procedures at their institution
- advise students of the placement testing process and the implications of placement into the mathematics curriculum
- assume a leadership role in the development of mathematics placement policies.

Departmental/institutional actions:

- assure that mathematics faculty are involved in the design of mathematics placement measures
- use multiple measures for initial placement of students into mathematics courses
- periodically determine the effectiveness of mathematics placement tests with an empirical study of cut-off scores and student success
- place students into the most advanced course appropriate for their program for which they have prerequisite skills.

Learning Styles

How students learn mathematics is influenced by their learning style, defined as "the preferences, tendencies, and strategies that individuals exhibit while learning."[5] An effective mathematics curriculum is one that provides students of every learning style an opportunity to engage in a topic, connect with the material, and then stretch their learning capacity in other learning modes.[6] Instructional strategies and activities that take cultural, personal interaction, and communication styles into consideration can also contribute to student success, particularly for new students and poorly prepared students, among whom most attrition occurs.[7]

> **… adults … need to be ready to learn in order to learn.**
>
> AMATYC, *Mathematics for the Emerging Technologies,* 2003, p. 9

Research in learning suggests that instructional approaches for children provide similar achievement results as those designed strictly for adults.[8] Though some assumptions about learning apply equally to adults and children, some researchers believe that adults and children learn differently.[9] There is evidence that adults tend to be more self-directed and prefer learning that is learner-centered,[10] than children, who have fewer experiences and pre-established beliefs. Instructors should choose methods that are appropriate for *their* students. The goal is to help each student understand why a mathematical concept is important to learn, how to navigate information to be learned, and how the topic relates to his/her experiences.[11]

In general, learning styles can be categorized according to different personality or cognitive trait characteristics. There are three sensory types of learning styles: auditory (*hearing*), tactile (*doing*), and visual (*seeing*). Other learning styles are extensions of these three basic styles. Most mathematics instructors are visual, abstract, and individual learners. Learning styles for developmental mathematics students are visual, tactile concrete, auditory, and social.[12] Students may exhibit all or many of the characteristics at any one time. Preferences for one style or another may be strong, moderate, or mild.

An individual's learning style can be identified using available instruments. Some students have a different learning style for mathematics than for other academic subjects, such as English or history. Therefore, to identify a student's mathematics learning style, it is important to identify and use an inventory specifically designed for mathematics.[13] Once a mathematics learning style has been identified, faculty can help students employ one or more of the strategies outlined in Table 3 to maximize their learning of mathematics.

Students and faculty need to work together to understand and address a student's learning style(s) and be open to trying multiple instructional strategies to maximize each student's learning in mathematics. Mathematics education should focus not only on reinforcing areas of student strengths, but also on effective learning strategies to enhance student performance in less dominant areas.[15] Professional development in how to use multiple approaches in instruction that address the complete range of learning styles should be provided to faculty. Once faculty are knowledgeable about learning styles and trained to respond to them, they often feel more comfortable using multiple instructional strategies in their classrooms. Students who discover, understand, and apply study skills and learning strategies to complement their learning style are more likely to become more efficient in learning mathematics and making sense of new information.

Table 3 Selected Learning Styles Characteristics and Strategies for Students[14]

Learning Style	Characteristics	Strategies for Students
Active/ Tactile/ Concrete	Retains and understands information as a result of doing something manual or involving the sense of touch.	• Use mathematics manipulatives as a concrete demonstration to make sense of a problem situation. • Draw a picture, make a table, build a physical model of a problem. • Have students act out a concept.
Active/ Social	Retains and understands information as a result of discussing or explaining to others.	• Participate in study groups. • Discuss concepts with the instructor and other students.
Analytic	Learns concepts and rules from experts.	• Listen to lectures. • Watch a demonstration.
Dynamic	Learns by exploring and looking for other possibilities for solving problems.	• Create and complete mathematics projects • Use trial and error to find mathematics patterns.
Global	Learns in large jumps, absorbs material randomly, is able to solve complex problems quickly and in novel ways.	• Relate new mathematics topics to previous knowledge.
Innovative	Learns mathematics by personally relating mathematics to himself/herself using feelings.	• Discuss mathematics ideas with others • Look for personal meaning in mathematics.
Intuitive	Discovers possibilities and relationships, is comfortable with abstractions and mathematical formulations, dislikes memorization and routine calculations.	• Seek interpretations and theories that provide proofs for theorems or formulas.
Reflective	Thinks about information quietly first and prefers to work alone.	• Incorporate reflection time as a part of study time. • Practice problems using computer software.
Sensing/ Common Sense	Learns facts by connecting concepts to real-world situations; prefers to see the usefulness and practical application of mathematics.	• Consult other sources for specific real-world examples of mathematics concepts and procedures • Seek hands-on learning experiences.
Sequential	Understands linear steps and follows logical paths to find solutions.	• Ask instructor to supply steps to solutions for problems.
Verbal	Prefers written and spoken explanations.	• Make summaries or outlines of course material. • Listen to classmates' explanations. • Read written explanations aloud. • Explain how to solve a problem to a tutor or classmate.
Visual	Remembers pictures, diagrams, flowcharts, formulas, and procedures.	• Seek diagrams, schematics, course material that can be viewed. • Create concept maps. • Color-code notes with highlighters. • Make flash cards with color coding.

Students will be expected to do the following:

- accept responsibility for their own learning
- apply strategies to complement their dominant learning style
- create, think, and reflect about mathematics concepts
- engage in multiple instructional and learning strategies to maximize their learning
- respect the different learning styles of other students.

Implementation recommendation: Students and faculty will be aware of different learning styles and implement supportive strategies to maximize student learning in mathematics.

Actions to support this recommendation

Faculty actions:

- understand student learning styles and become aware of one's own learning style
- help students identify their mathematics learning style(s)
- implement multiple instructional strategies to address multiple learning styles.

Departmental/institutional actions:

- provide academic resources to develop and support multiple instructional strategies
- provide professional development opportunities on learning styles for mathematics faculty and student support staff.

Learning to Be Problem Solvers

Becoming an efficient, independent problem solver should be a goal of every mathematics student. But for many students, mathematics is viewed as a "string of procedures to be memorized, where right answers count more than right thinking."[16] Authentic problem solving does not necessarily involve memorizing procedures and usually involves being motivated to solve the problems.[17]

> **…a teacher of mathematics has a great opportunity. If he fills his allotted time with drilling his students in routine operations he kills their interest, hampers their intellectual development, and misuses his opportunity…if he challenges the curiosity of his students…and helps them to solve their problems with stimulating questions, he may give them a taste for, and some means of, independent thinking.**
>
> George Polya, *How to Solve It,* 1973, p. v

> *"…good problem-solving behavior usually is not fostered by having students imitate how teachers solve problems. Because teachers typically demonstrate only correct moves, students often come to view problem solving as that of delving into a mysterious bag of tricks to which only a select few are privy."*[18]

To build problem-solving skills, faculty need to engage students actively in the learning process, create opportunities for exploration, and help them recognize that there may not be a rule to memorize or algorithm to follow for a given problem.

Expert problem solvers have access to rich, well-connected knowledge of mathematical concepts and possess confidence following a long history of successful problem solving. They also have an ability to imagine and conjecture possible solution paths, to monitor their progress and dynamically revise or abandon solution paths, and to verify that a solution is reasonable and makes sense. In contrast, developmental mathematics college students rarely plan a solution in advance, may demonstrate an inability to consistently monitor their progress, and have varying degrees of success recognizing that a solution attempt is not progressing toward the desired goal.[19] When their initial strategy is not productive, these students have difficulty switching to an alternative strategy. Faculty and students need to take these characteristics into consideration and employ and engage in classroom activities that focus on boosting students' confidence and building a reservoir of problem-solving strategies. When students are given opportunities to use multiple approaches to solve problems, they come to recognize that mathematics is more than computation or getting the single right answer—it is a balance of process and product—a combination of good thinking and meaningful answers.

Students will be expected to do the following:

- recognize that problem solving is an essential skill to be developed or improved in any mathematics course
- understand and use a variety of problem-solving strategies.

Implementation recommendation: Faculty will be intentional and persistent throughout every mathematics course in helping students improve their problem-solving skills.

Actions to support this recommendation

Faculty actions:

- plan and model classroom experiences using multiple problem-solving approaches
- provide students with adequate time for planning, monitoring, reflecting, and understanding multiple approaches to solving problems.

Departmental/institutional actions:

- provide professional development for faculty and tutors to learn how to incorporate multiple problem-solving instructional strategies into their presentations.

Mathematics Anxiety and Other Factors That Influence Learning

The beliefs and attitudes that students bring with them to the classroom play a major role in how they learn mathematics. Some students believe that mathematics is about computation and that they are to find *the* correct answer in five minutes or less. They believe that they are to be passive in the learning process. They may also view mathematics as a collection of rules, facts, skills, and algorithms that need to be memorized. Some students think that mathematics is meaningless when there is a lack of context.[20] College mathematics students often believe that they should accept procedures without trying to understand the concepts, because they feel they are not capable of creating mathematics themselves.[21]

Attitudes toward mathematics can create either a feeling of confidence or anxiety that may have a positive or negative effect on mathematical behavior.[22] "Math anxiety" is described as a feeling of dread that is experienced when a person attempts to understand and solve mathematics problems. Factors such as age or maturational level, relationship between student and teacher, and the nature of the learning environment, including instructional methods used and learning resources available, influence why mathematics anxiety occurs.[23] Prior experiences in mathematics often play an important role. This anxiety is a major concern for many college students, particularly females and those with weak mathematics backgrounds.

> **Your goal…should be to guarantee every student math mental health, namely, the willingness to learn the math they need when they need it.**
>
> Sheila Tobias, Math Mental Health, *College Teaching* 39(3), 1991, p. 93.

Depending on the degree of mathematics anxiety, the student's fears can develop into "learned helplessness," the belief that one is unable to do mathematics at all.[24] Learning style issues can increase anxiety. A student who is predominantly a tactile learner may feel bewildered when mathematics is presented as strictly symbol manipulation. As a result, faculty and students should work together to identify mathematics anxiety and manage the learning process. Strategies for students to cope with mathematics anxiety can be grouped into four categories as noted in Table 4. All four categories are beneficial, but approach strategies have been shown to be most successful.[25] Faculty can assist students in overcoming and managing their anxiety by suggesting that students engage in one or more of the following actions.

Table 4 Strategies for Coping with and Helping to Alleviate Mathematics Anxiety[26]

Strategy	Student Actions
Approach strategies involve the active learning of mathematics.	• Complete homework on time so as not to fall behind. • Maintain a regular study schedule and set aside extra study time before examinations. • Ask questions in class. • Talk with the instructor outside of class when concepts are unclear. • Form study groups with other students. • Visit a mathematics resource center or tutoring center for extra help. • Use supplemental resources, books, Web sites, or computer-based instruction.
Avoidance strategies involve taking a temporary break from studying mathematics to alleviate distress.	• When frustrated, maintain positive attitudes and remind themselves that they are good students. • Participate in sports or exercise to relieve stress.
Social support strategies involve sharing common experiences with others.	• Discuss mathematics concepts and experiences with other students. • Discuss mathematics concepts and experiences with the instructor, counselor, or advisor.
Relaxation strategies involve reducing stress through relaxation and self-control techniques.	• Practice systematic relaxation techniques such as deep breathing, tensing and relaxation, and visualization. • Practice long-term relaxation techniques. • Replace negative self-talk during homework or test-taking with positive self-talk.

Students will be expected to do the following:

- play an active role in the learning of mathematics
- understand that learning often involves effort, frustration, and struggle
- understand the impact of mathematics anxiety on their learning
- employ a variety of strategies to cope with and alleviate mathematics anxiety.

Implementation recommendation: Students, faculty and support staff will understand the influence of students' attitudes toward learning mathematics and employ strategies to help alleviate mathematics anxiety, build confidence in solving problems, and maximize student learning in mathematics.

Actions to support this recommendation

Faculty actions:

- be aware of the diverse mathematics backgrounds of their students and be sensitive to the impact that mathematics anxiety has on students
- answer questions and explain material carefully and clearly
- assign and review homework regularly to provide periodic and timely feedback

- be patient, supportive, and available to help when students are frustrated or confused
- refer students to appropriate support services for help in reducing mathematics anxiety
- be sensitive to the fact that students' family and job responsibilities may occasionally impact on their ability to complete course requirements
- use multiple assessment measures.

Departmental/institutional actions:

- offer mathematics study skills workshops for students
- provide a sufficient number of qualified, well-trained tutors who are available during the day, evening, and weekend
- provide training for counselors and other student support staff to address students' mathematics anxiety.

Inside and Outside the Mathematics Classroom

The physical environment of the classroom and the support services available outside of the classroom influence student success in mathematics. Every classroom develops its own social characteristics.[27] The institution needs to create the best environment for the learning of mathematics and support the characteristics of social interaction. The classroom layout, furniture, and the ease of bringing technology into the classroom all contribute to the learning of mathematics. As an example, classroom lighting needs to be flexible to support the use of technology and may require multiple light switches and the ability to dim the lights.

While most formal mathematics instruction takes place in a physical classroom, the boundaries and practices of the traditional classroom are being redefined. In the virtual classroom of distance education, the roles of the student and faculty are changing. Because students and faculty may be separated by time and location, students need to accept even more responsibility for their learning. In turn, faculty need to be even more available and responsive to students, utilizing alternative communication tools such as voice mail, e-mail, fax, electronic blackboards, and discussion groups.

Communication of mathematical ideas can extend beyond the classroom door. Learning "occurs within the broader social system that pervades the campus."[28] Thus, mathematics departments need to look for ways to encourage faculty-student and student-student interaction outside of the mathematics through mathematics clubs, local internships, speakers, mathematics-related field days, mathematics contests, study groups, and peer tutoring.

> **...the classroom environment communicates subtle messages about what is valued in learning and doing mathematics.**
>
> National Council of Teachers of Mathematics, *Principles and Standards for School Mathematics,* 2000, p. 18

Students can also benefit from a college tutoring center or mathematics resource center, staffed by qualified, trained tutors who themselves are students or employees of the college. Successful mathematics centers provide the following:

- multiple and varied resources for students
- peer and professional tutoring
- computers and other technology that supplement instruction
- student workshops focusing on learning styles and reducing mathematics anxiety
- opportunities for students to work individually and in groups
- adequate space for mathematics tutoring.

An academic testing center can supplement classroom activities by offering out-of-class testing, make-up testing, and placement testing. From the classroom to the testing center, from the math club to the tutoring center, a successful environment is the result of careful planning and input from mathematics faculty and cooperation and commitment of everyone within the institution.

Students will be expected to do the following:

- use mathematics learning resources available outside of the classroom
- work with other students, tutors, and faculty outside of the classroom.

Implementation recommendation: Institutions will provide appropriate physical facilities and academic support resources to promote student success in mathematics and complement learning experiences.

Actions to support this recommendation

Faculty actions:

- be involved in the design of and decision-making about physical spaces that support mathematics instruction
- identify and recommend needed technology
- encourage interaction with students and between students inside and outside of the classroom
- be available outside of the classroom to assist individual students.

Departmental/institutional actions:

- supply the necessary equipment, including technology, to create classroom environments and mathematics learning centers that maximize the learning of mathematics
- provide adequate space and resources for peer and professional tutoring and mathematics resource centers
- provide adequate office space, computers, and access to student data for full-time and adjunct faculty.

Conclusion

Students, faculty, and institutions play important roles in creating productive learning environments to learn mathematics. Strategies for addressing initial placement into mathematics courses, the needs of the diverse student population, differences in student learning styles and problem solving skills, the impact of mathematics anxiety, ever-changing instructional formats, and planning for facilities and resources are best managed when all stakeholders actively participate and are guided by research. Mathematics faculty should set high academic goals for all students, complemented by programs and processes that help students achieve those goals.

The Implementation Standard for Student Learning and the Learning Environment, along with the recommendations presented in this chapter, can help to put the Standards for Intellectual Development from *Crossroads in Mathematics* into practice. Students *can* be successful in mathematics when students, faculty, and institutions collaboratively develop strategies, constructively confront the changing mathematics environment, and work to implement the actions in this and the following chapters. However, institutions need to provide the academic resources for faculty and student support personnel necessary to sustain and improve learning in mathematics. The results of research summarized in this chapter, as well as lessons learned by experienced practitioners, provide guidance to students, faculty, departments, and institutions for improving mathematics instruction in the first two years of college.

HIGHLIGHTS Implementing the Standard for Student Learning and the Learning Environment

Student Learning and the Learning Environment

Faculty and their institutions will create an environment that optimizes the learning of mathematics for all students.

At a standards-based institution, the *faculty*

- ☑ clearly define high expectations and communicate these to all students.
- ☑ are actively involved in understanding and recommending improvements to institutional policies for mathematics placement.
- ☑ use a variety of instructional methods to address the learning styles of all students.
- ☑ play an active role in planning and creating the learning environment.

At a standards-based institution, the *mathematics department* and the *institution*

- ☑ provide a learning environment that supports, values, and affirms the diverse needs of all learners.
- ☑ provide facilities, tutors and staff for mathematics resource centers.
- ☑ use multiple measures to place students in mathematics courses.
- ☑ provide professional development regarding learning styles, mathematics anxiety, and multiple problem-solving strategies so that faculty and instructional staff have the skills needed to address the diverse student population and dynamic curriculum.
- ☑ design and equip classrooms that encourage active learning and use of technology.

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1 Marwick, J. (2002). Charting a Path to Success: How Alternative Methods of Mathematics Placement Impact the Academic Success of Community College Students. *Community College Enterprise*, 8(2), pp. 41–50.

2 Apter, J. R. (1975). *The Development of a Community College Mathematics Placement Examination*. Unpublished doctoral dissertation, University of Illinois at Urbana-Champaign, Urbana, IL.

Armstrong, W. B. (1999). *Explaining Community College Outcomes by Analyzing Student Data and Instructor Effects*. Unpublished doctoral dissertation, University of California Los Angeles, Los Angeles, CA.

Robinson, S. H. (1998). *An Analysis of Placement Systems for New and Returning Community College Students, Specifically in College Preparatory and Entry-Level College-Level Mathematics Courses*. Unpublished doctoral dissertation, University of Central Florida, Orlando, FL. Dissertation Abstracts International, pp. 59–63, A0707.

3 Lewis, L., Farris, E., & Greene, B. (1996). *Remedial Education at Higher Education Institutions in Fall 1995*. Washington, DC: National Center for Education Statistics. (Statistical analysis report NCES 97–584).

4 Grubb, W. N., Worthen, H., Byrd, B., Webb, E., Badway, N., Case, C., Goto, S., & Villeneuve, J. C. (1999). *Honored But Invisible: An Inside Look at Teaching at Community Colleges*. New York, NY: Routledge, p. 5.

5 Thomson, B. S. & Mascazine, J. R. (1997). *Attending to Learning Styles in Mathematics and Science Classrooms*. ERIC Digests. Columbus, OH: ERIC Clearinghouse for Science, Mathematics and Environmental Education. Retrieved 4/3/2006 from http://www.ericdigests.org/2000-1/attending.html, p. 1.

6 Brown, B. L. (2003). Teaching Style vs. Learning Style. *Myths and Realities*, No. 26, ACVE Publications, p. 2. ERIC document ED 482329.

7 Sims, R. R. & Sims, S. J. (1995). *The Importance of Learning Styles: Understanding the Implications for Learning, Course Design, and Education*. Contributions to the Study of Education, No. 64. Westport, CT: Greenwood Press, p. 76.

Claxton, C. S. & Murrell, P. H. (1987). Learning Styles: Implications for Improving Educational Practices. *ASHE-ERIC Higher Education Report No. 4*. Washington, DC: Association for the Study of Higher Education.

8 Rachal, J. R. (1994). *Andragogical and Pedagogical Methods Compared: A Review of the Experimental Literature* (Informational Analysis). Hattisburg, MI: University of Southern Mississippi. ERIC document ED380566.

9 The term "andragogy" was defined as "the art and science of helping adults learn" by M. Knowles. Knowles, M. (1984). *The Adult Learner: A Neglected Species*. Houston, TX: Gulf Publishing. Retrieved 4/3/2006 from http://agelesslearner.com/intros/andragogy.html.

10 Harriman, G. *E-Learning Resources*. Retrieved 4/3/2006 from http://www.grayharriman.com/adult_learning.htm.

11 Conner, M. L. Andragogy and Pedagogy. *Ageless Learner*, 1997–2004. Retrieved 4/3/2006 from http://agelesslearner.com/intros/andragogy.html.

[12] Nolting, P. (2005). *Learning Styles of Developmental Math Students*. Title III Research. Retrieved 4/3/2006 from www.mccfl.edu/pages/2158.asp. *Comparing Student Learning Styles of Developmental Math Students to Faculty Learning Styles*. Title III Research. Retrieved 4/3/2006 from www.mccfl.edu/pages/2160.asp.

[13] Myers-Briggs Type Indicator (Briggs, Myers & McCaulley, 1985), The Kiersey Temperment Sorter (Kiersey & Bates, 1984), Dunn Model (Dunn, 1996), The Kolb Learning Style Inventory for Mathematics (Kolb, 1976), Felder-Silverman Index of Learning Styles (Felder, 1993).

[14] Felder, R. M. (1993). Reaching the Second Tier: Learning and Teaching Styles in College Science Education. *Journal of College Science Teaching*, 22(5), pp. 286–290.

Nolting, P. (2002). *Winning at Math: Your Guide to Learning Mathematics through Successful Study Skills*. Bradenton, FL: Academic Success Press, Inc.

[15] Sims & Sims (1995).

[16] National Research Council (1989). *Everybody Counts: A Report to the Nation on the Future of Mathematics Education*. Washington, DC: National Academy Press, p. 10.

[17] Kloosterman, P. & Stage, F. (1992). Measuring Beliefs about Mathematical Problem Solving. *School Science and Mathematics*, 92(3), p. 109–115.

[18] Lester, F. (1983). *Trends and Issues in Mathematical Problem-Solving Research*. In R. Lesh and M. Landau (Eds.). *Acquisition of Mathematics Concepts and Processes*. New York, NY: Academic Press, Inc., p. 229.

[19] Vicich, J. (2002). *Mathematical Problem-Solving Behaviors of Undergraduate Developmental Algebra Students*. Unpublished doctoral dissertation, Arizona State University, Tempe, AZ.

[20] Carlson, M. (1997). Obstacles for College Algebra Students in Understanding Functions: What Do High-Performing Students Really Know? *AMATYC Review*, 19(1), pp. 48–59.

Schoenfeld, A.H. (1985). *Students' Beliefs about Mathematics and Their Effects on Mathematical Performance: A Questionnaire Analysis*. ERIC document ED259950.

Spangler, D. A. (1992). Assessing Students' Beliefs about Mathematics. *The Mathematics Educator*, 3(1), pp. 19–23.

Yusof, M. & Tall, D. (1999). Changing Attitudes to University Mathematics through Problem Solving. *Educational Studies in Mathematics*, 37, pp. 67–82.

[21] Schoenfeld, A. (1988). When Good Teaching Leads to Bad Results: The Disasters of "Well Taught" Mathematics Courses. *Educational Psychologist*, 23(2), pp. 145–166.

[22] Schoenfeld, A. (1987). *What's All the Fuss about Metacognition?* In A. H. Schoenfeld (Ed.), *Cognitive Science and Mathematics Education*. Hillsdale, NJ: Lawrence Erlbaum Associates, pp. 189–215.

De Corte, E. & Op't Eynde, P. (2003). *Students' Mathematics-Related Belief Systems: Design and Analysis of a Questionnaire*. Paper presented at the Annual Meeting of the American Educational Research Association, Chicago, IL, 2003. ERIC document ED475708.

[23] Fiore, G. (1999). Math-Abused Students: Are We Prepared to Teach Them? *Mathematics Teacher*, v. 92, pp. 403–406.

Arem, C. (2003). *Conquering Math Anxiety: A Self-Help Workbook*. Pacific Grove, CA: Brooks-Cole.

[24] Tobias, S. (1993). *Overcoming Math Anxiety: Revised and Expanded*. New York, NY: W. W. Norton & Co.

[25] Peskoff, F. (2001). Mathematics Anxiety and the Adult Student: An Analysis of Successful Coping Strategies. In Schmitt, M. J. & Safford-Ramus, K. (Eds.), *Adults Learning Mathematics—7: A Conversation Between Researchers and Practitioners*. Proceedings of ALM—7, July 6–8, 2000, at Tufts University, Cambridge, MA: National Center for the Study of Adult Learning and Literacy, Harvard Graduate School of Education.

[26] Peskoff (2001). Nolting (2002).

[27] Yackel, E. & Rasmussen, C. (2002). *Beliefs and Norms in the Mathematics Classroom*. In G. Leder, E. Pehkonen, & G. Toerner (Eds.), *Beliefs: A Hidden Variable in Mathematics Education*. Dordrecht, The Netherlands: Kluwer, pp. 313–330.

[28] Tinto, V. (1997). Classrooms as Communities: Exploring the Educational Character of Student Persistence. *The Journal of Higher Education*, 68(6), p. 619.

Chapter 5

Assessment of Student Learning

Assessment is a comprehensive process with many facets. This chapter focuses exclusively on assessment of student learning of mathematics, one of the most important professional responsibilities of mathematics faculty and one that should be as much a part of the faculty role as teaching classes.[1] Assessment, because it involves evaluating student performance, extends beyond assigning grades. While grades are important, they do not always tell us exactly what students know and can do. Two students who receive the same grade may not know the same concepts or have the same skills. Grades may be a way to rank students' achievement, but are not indicators of specific knowledge or abilities. Standards for grading may differ with each instructor.[2] Thus, grades do not generally provide the necessary information to make needed changes in pedagogy, curriculum, prerequisites, or other aspects of courses or programs.

Assessment of student learning is a process of helping mathematics faculty adapt instruction to the needs of students. Assessment provides the mathematics department with information to make informed decisions about course content. It is also the process by which a college assesses what mathematics students know at the end of a student's course or program. Assessment is an ongoing activity that leads to improvement in student learning by providing data necessary for making informed decisions at the class, course, and program levels.

Implementation Standard: Assessment of Student Learning

Mathematics faculty will use results from the ongoing assessment of student learning of mathematics to improve curriculum, materials, and teaching methods.

The Assessment Cycle

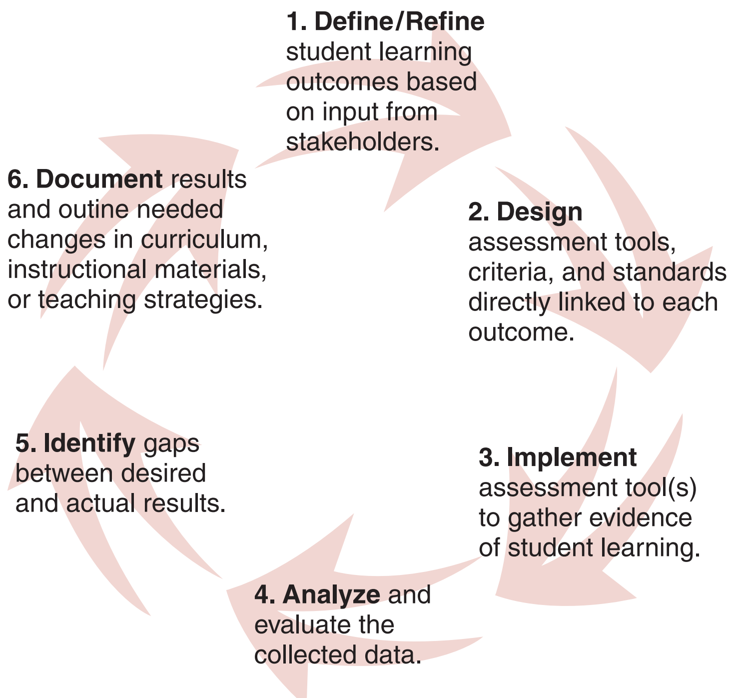
Assessment is a cycle that begins with a statement of desired student learning outcomes. The Implementation Cycle presented in Chapter 3 is easily adapted to the assessment of student learning in Figure 2 on the next page. The first step involves defining learning outcomes—clear statements of what students will know and be able to do after they have finished a class, course, or program.[3] Learning outcomes in classes, courses, and programs should reflect the skills, knowledge, and ways of thinking the mathematics department believes are important for students to learn. Assessment efforts should not be limited to what is easy to measure.[4]

In step 2, tools are designed to assess student learning relative to the stated outcomes. Assessment must be tied to specific learning outcomes in order to be effective. A standard is established against which each outcome will be judged. For example, if an outcome requires students to communicate their mathematical thinking, assessment tasks that require written explanations or verbal presentations may be appropriate.

In steps 3–5, the assessment tool is implemented and data are collected and analyzed using the established criteria for each learning outcome. Gaps between desired and actual results are identified and discussed among mathematics faculty. This thoughtful discussion about what is most important about student learning in mathematics leads to action plans for improvement in the assessment tool, the learning outcomes, course materials, instructional methods, course prerequisites or other curricular or policy changes.

Finally in step 6, results are documented and improvements are implemented based on the analysis of the data. This final step of the cycle is often referred to as the feedback loop. Then the Assessment Implementation Cycle begins again.

Figure 2 The Assessment Implementation Cycle



Assessment at Three Levels

The assessment cycle provides assurances to students, colleagues, and the external community about the content and quality of mathematics in the first two years of college. Effective assessment includes the documentation of student learning at three levels: class, course, and program. Assessment at each level is the responsibility of all faculty. It should reflect not only student knowledge of facts and procedures, but also critical thinking and ways of thinking about and communicating mathematics. When faculty begin an assessment, they should start on a small scale, choosing one or more course or program outcomes to assess. As faculty gain facility with the assessment process, other learning outcomes can be added to the assessment plan.

> **Higher quality teaching is grounded in a careful and thorough alignment of curriculum, assessment, and high standards for student learning.**
>
> U.S. Department of Education, *Before It's Too Late: A Report to the Nation from the National Commission on Mathematics and Science Teaching for the 21st Century,* 2000, p. 22.

Classroom assessment involves individual instructors assessing individual students' learning outcomes with instructor-developed tools. Many assessments are used throughout the term and changed frequently. Students are informed often about their progress. Individual faculty who use classroom assessment techniques discover how to adapt instruction to address the learning needs of individual students in a specific mathematics class. Faculty make frequent and immediate changes to class activities and methods based on assessment results.

Assessment efforts at the course level provide evidence of student learning and motivate changes beyond the individual classroom. Learning outcomes in individual mathematics classes, in all sections of a particular course, or in sequences of courses or specific programs, are assessed to determine if students are meeting agreed-upon course learning outcomes. A group of faculty teaching intermediate algebra, for example, can collaborate to discover the instructional strategies or materials best suited to help all students learn the relationship between a quadratic equation and its corresponding graph, an agreed-upon course outcome.

Assessing outcomes at the program level is the broadest and most overarching form of assessment discussed in this chapter. Faculty should develop a consensus about the essential student learning outcomes for mathematics courses and programs, as well as the college's quantitative literacy general education outcomes. Faculty can use the results of program assessment to determine how all courses in a program, mathematics and nonmathematics courses, help students achieve quantitative goals and objectives. In addition, external accrediting agencies may have an impact on mathematics outcomes within programs.

Although classroom assessment should be ongoing, course and program outcomes may be assessed at regular intervals with faculty choosing a different course or program each year. It is important, however, to complete the assessment cycle for the learning outcomes chosen. In addition to collecting data about outcomes, improvements should be implemented based on an analysis of that data. The following table outlines some of the differences in assessment practices at the three levels.

Table 5 Multiple Levels of Assessment

	Classroom level	Course level	Program level
Who assesses?	Individual instructors	A group of mathematics faculty who teach a course	Mathematics faculty, possibly as part of an interdisciplinary committee
What is assessed?	Individual students' learning outcomes	Course outcomes (via a representative group of students)	Program outcomes (via a representative group of students)
Where?	Individual classes	All sections (or a representative sample) of a given course	At the completion of a degree or program via a representative group of students
How?	Instructor-developed assessment tools and rubrics	Department-developed tools and rubrics	College-developed tools and rubrics
How many assessments?	Many assessments that can be frequently changed to give detailed feedback to students	Fewer assessments that remain fairly consistent over time	Fewest assessments that remain fairly consistent over a longer period of time
Levels of measurement?	Many levels to differentiate fine differences in expectations, performance	Three levels: exceeds expectations, meets expectations, does not meet expectations	Three levels: exceeds expectations, meets expectations, does not meet expectations
Potential actions?	Changes made immediately to class activities, methods, or materials	Changes made to course curriculum, instructional methods, or materials	Changes made to program curriculum, prerequisites, instructional methods, or materials

Classroom Assessment

Classroom assessment begins with an individual instructor outlining student outcomes for a particular class. By continuing through the assessment cycle and the feedback loop, student progress and growth in mathematics understanding can be assessed, along with mastery of basic skills. Good teachers have always monitored student learning, but assessment makes this monitoring more systematic.[5] Students are observed in the process of learning by collecting frequent feedback on what, how much, and how well they are learning. In addition to enhancing students' learning, classroom assessments positively affect students' perceptions and attitudes about the learning process.[6] Individual instructors can test assumptions and impressions by checking them against student performance before the student completes the class and can make adjustments before the end of the term. "When classroom assessment is fully integrated into a course, ongoing assessment becomes part of the faculty member's overall plan for instruction."[7]

Classroom assessments, often unique to an individual instructor, are activities that are integrated into instruction and administered during a lesson or class. These assessments may or may not be used for grading purposes, and provide both the instructor and the student with valuable feedback about each student's progress relative to the learning outcomes. Student demonstrations to the class, discovery-oriented activities performed in groups, one-minute papers, or other classroom assessment techniques may be included. Assessment that enhances mathematics learning should become a routine part of ongoing classroom activity and should not be viewed as an interruption.[8] In this way, assessment is not a single event, but integrated with teaching and learning.

> **Through practice in classroom assessment, faculty become better able to understand and promote learning, and increase their ability to help the students themselves become more effective, self-assessing, self-directed learners.**
>
> Thomas Angelo and Patricia Cross, *Classroom Assessment Techniques, A Handbook for College Teachers,* 1993, p. 4

Classroom assessment can have a positive impact on student behavior and performance when it focuses on improving the learning experience, not on identifying individual student weakness. The assessments may be anonymous and results may be analyzed in the aggregate. Sharing assessment data in class can be an effective learning exercise.[9] For example, a statistics class might analyze student responses to a brief in-class survey or error patterns in a problem set, then discuss what kinds of improvements might be made. Student self-assessment can provide an opportunity for students to reflect upon their responsibility for their own learning. When faculty respond to assessment results by sharing their reasons for changes in instruction, student motivation is increased. Students realize faculty are interested in their success as learners.

Classroom assessments can enhance student performance by requiring students to participate actively, reinforcing their grasp of course material, and participating in their own self-assessment. The results are used immediately to redirect the learning experience and to address difficulties. Research suggests that listening to students, asking them appropriate questions, and giving them the opportunity to show what they know in a variety of ways are effective strategies that increase student learning.[10]

Students will be expected to do the following:

- engage in regular reflection and self-assessment of their performance.

> **Implementation recommendation:** Each faculty member will use multiple classroom-assessment techniques as an integral part of instruction to assess student learning and use those results to adjust instructional methods and materials.

Actions to support this recommendation

Faculty actions:

- incorporate classroom assessment activities into class activities on a regular basis
- provide feedback at times and in ways that are most helpful to students
- adjust classroom activities in response to assessment information
- discuss assessment results with students and explain how the information is being used to make instructional decisions.

Departmental/institutional actions:

- support faculty in their assessment efforts in the mathematics classroom
- provide professional development and training in assessment techniques for mathematics faculty
- provide support in and out of the mathematics classroom for implementation of recommendations based on assessment results.

Course Assessment

Course assessment extends class assessment by involving the department collectively. This type of assessment, which begins by defining assessment terms and broadly communicating these definitions to mathematics faculty and the college community, remains fairly constant over time. Mathematics faculty should first agree upon the core student learning outcomes for each mathematics course. Course outcomes are the same for all sections of a given course, while individual instructors may include additional learning outcomes in class syllabi. Courses taught using a variety of instructional modalities should use equivalent assessment tools to ensure student learning is occurring at a high level regardless of the method of instructional delivery.[11] A focus on competencies is the bridge between using the traditional measure of class-time and using new and varied methods of instructional delivery as the determining factor to award credit hours. When developing learning outcomes, attention must be given to reasoning skills and ways of thinking about and communicating mathematics, in addition to necessary learning outcomes about facts and procedures.

All course outcomes should be communicated to students in each section of the course at the beginning of the term. Each instructor should refer to the outcomes as the term progresses so that students can evaluate their own learning relative to the stated course outcomes. A course-based assessment may be administered at the end of a term by all faculty teaching the course. Other assessment formats may be equally effective. Assessment instruments should directly measure student performance relative to one or more course learning outcomes.

> **The quality of teachers' instructional decisions depends, in part, on the quality of their purposeful sampling of evidence during instruction.**
>
> National Council of Teachers of Mathematics, *Assessment Standards for School Mathematics,* 1995, p. 45.

Data gathered from all course sections, or a representative sample, should be aggregated and analyzed. Care must be taken to present assessment results without linking specific class section results to specific instructors. Course assessment should be used to assess overall student learning of course outcomes, not to evaluate individual instructors. Individual instructors should compare their students' performance to that of the department to better understand the relationship between specific classroom materials and methods, and student learning. Faculty in the mathematics department as a whole should reflect upon and discuss the assessment results, materials, and processes leading to those results. This important step in the course assessment process is where best practices are discussed and where improvements to a course are planned.

Data should also be disaggregated to compare results of diverse student subpopulations. It is important to monitor the learning of various subpopulations and to consider any differences that occur in order to develop strategies to address the cause of any learning deficiencies. Similarly, it may be useful to analyze the characteristics that distinguish the top quartile of learners from the lowest quartile.

If students as a group are not achieving the desired course outcomes, faculty should respond in one or more ways to implement the agreed upon changes and begin the cycle again. This may include reexamining course content or prerequisites or assessment tools, altering teaching methods or activities, changing the type and frequency of testing, adjusting the percentage of group work or lecture, or modifying the amount of feedback or individual attention given to students. Course assessment provides a systematic process to enact these improvements and a structure that encourages faculty to reflect upon and discuss what best helps students learn mathematics course content.

Students will be expected to do the following:

- be aware of and focus their study on achieving learning outcomes.

Implementation recommendation: Mathematics departments will determine outcomes for each course and measure student learning for all students enrolled, relative to these outcomes.

Actions to support this recommendation

Faculty actions:

- agree upon the core student learning outcomes for each mathematics course
- communicate course outcomes to students at the beginning of each term.

Departmental actions:

- involve full-time and adjunct faculty in designing and implementing course-based assessment
- link department-wide assessment instruments to course outcomes
- assess courses using a representative group of students
- plan for and conduct periodic assessment of all mathematics course outcomes
- analyze assessment data and use the results to improve student learning.

Program Assessment

Assessment of student learning outcomes at the program level is an important complement to indirect assessments such as student retention and unit cost studies that are generally part of a program review. Program assessment has three components: (1) assessment of mathematics programs; (2) assessment of other academic programs that include mathematics learning outcomes; and (3) assessment of the mathematics component of the college's general education outcomes.

The first component of assessment of the mathematics program refers to the assessment of a sequence of mathematics courses. For example, mathematics departments should assess the algebra sequence from prealgebra through college algebra, the calculus sequence, and the sequence of mathematics courses required for business majors or prospective teachers. The outcomes for all courses within the mathematics department should clearly illustrate the relationship among the various mathematics courses and what students should learn in each course. Course outcomes in sequential or prerequisite courses should be designed so important concepts are learned well and unnecessary review is eliminated. If the curriculum, the courses, and the process of learning are integrated, gaps in students' learning are minimized.[12] Assessment of student learning in developmental mathematics is especially important because those courses are prerequisites to many other courses. Assessment in the developmental mathematics program should measure the quantitative literacy and other mathematics skills necessary for student success in future college-level courses.

> **Program assessment is an ongoing process designed to monitor and improve student learning. Faculty develop explicit statements for what students should learn, verify that the program is designed to foster this learning, collect empirical data that indicate student attainment, and use these data to improve student learning.**
>
> Mary J. Allen, *Assessing Academic Programs in Higher Education*, 2004, p. 5

The second component in program assessment involves assessing mathematics outcomes in nonmathematics courses and programs. Since student success in many academic programs is directly linked to student learning in mathematics, mathematics faculty should collaborate with faculty in other programs. An analysis of results from the assessment of student learning at the end of such programs should be shared so faculty can collaborate to improve curriculum and instruction. Just as outcomes must be aligned in sequences of mathematics courses, so must outcomes be aligned between prerequisite mathematics courses and courses in other disciplines to give students the best opportunity to achieve academic goals. In this process, faculty may also discover that the mathematics content of a prerequisite course needs to be adjusted.

Mathematics faculty should also provide leadership for the third component of program assessment—the assessment of quantitative literacy at the college level. Mathematics skills and processes, mathematical modeling, and problem solving should be taught across the curriculum. If students are to be quantitatively literate, they must have opportunities to practice this in several contexts and in increasing sophistication throughout their learning experience. Data from program assessment may reveal that quantitative thinking is, in fact, not integrated across the curriculum, or that the problem-solving skills expected of our students upon graduation is different from the problem solving they practice while enrolled in classes.

Assessment tools for program assessment may include portfolios containing examples of student work linked to specific program outcomes, tests administered in the final course in the sequence, or cross-sectional samples of student work collected in "key courses" taken by all students enrolled in the program. Rubrics are often developed to describe and scale levels of student achievement on specific performance tasks.[13] An interdisciplinary capstone course or student electronic portfolios may be an effective means of assessing college-wide general education outcomes, providing a forum to identify gaps between desired and actual student learning, and showing students the connections between disciplines.

Program assessment requires that faculty work through the Assessment Implementation Cycle and seek reasons when students do not achieve desired mathematics outcomes. It is important to implement the improvements, which may involve course alignment, changes in prerequisites, or increased mathematics requirements, and proceed through the cycle again. Program assessment can also identify problems in structures, such as placement processes, that can serve as barriers to learning mathematics.

Implementation recommendation: Mathematics faculty, in collaboration with faculty in other departments, will design an assessment process to measure and improve student learning of mathematics and quantitative literacy in all academic programs.

Actions to support this recommendation

Faculty actions:

- identify assessment tools linked to desired student learning outcomes and proceed through the Assessment Implementation Cycle to implement improvements
- participate in the development and assessment of general education outcomes in mathematics
- determine which of the general education outcomes are met by completing a mathematics course.

Institutional actions:

- encourage collaboration among departments regarding instruction and assessment of mathematics outcomes embedded in nonmathematics courses
- implement periodic reviews and redesign of student learning outcomes in mathematics.

Conclusion

Good assessment practices lead to faculty reflection and action to improve student learning in mathematics. Assessment must occur at the class, course, and program levels of instruction and across different instructional and delivery models. All assessment efforts must be linked to student learning outcomes and assessment tools must be designed to measure what faculty believe is important for students to learn. When analyzing data, unfavorable results should be considered, as well as favorable results. Realizing a problem exists (that students are not achieving an outcome) is the first step in motivating faculty to consider changes. The most important result of assessment efforts is the discussion and introspection among mathematics faculty about what is really important for students to learn and how best to help them learn it.

Assessment plays a key role in both the scholarship of teaching and learning and professional growth. Educational assessment must be aligned not only with the design of thoughtful curricula and instructional strategies, but also with assessment practices that measure student learning relative to stated course and program outcomes. Assessment is integral to all that educators do and is important to the implementation of the principles and standards of *Beyond Crossroads*.

HIGHLIGHTS Implementing the Standard for Assessment of Student Learning

Assessment of Student Learning

Faculty will use results from the ongoing assessment of student learning of mathematics to improve curriculum, materials, and teaching methods.

At a standards-based institution, the *faculty*

- participate in ongoing assessment activities at the class, course, and program levels.
- design assessment tools that will elicit student work on the meaning and application of mathematics in addition to an understanding of mathematics skills and concepts.
- use multiple measures of student performance and a variety of assessment formats.
- reflect and act upon assessment results by completing the feedback loop to implement improvements in the process.

At a standards-based institution, the *mathematics department* and the *institution*

- use common assessment tools across course sections and different instructional formats.
- implement the assessment of quantitative literacy leveling each course and program.
- provide administrative support for faculty assessment efforts and for the implementation of recommendations.

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1 Lopez, C. L. (1999). A Decade of Assessing Student Learning: What We Have Learned? What's Next? Retrieved 1/16/2006 from www.ncahigherlearingcommission.org/annualmeeting/archive/index.html.

2 Walvoord, B. E. (2004). *Assessment Clear and Simple. A Practical Guide for Institutions, Departments, and General Education*. San Francisco, CA: Jossey-Bass.

3 Stiehl, R. & Lewchuk, L. (2002) *Outcomes Primer. Second Edition*. Corvallis, OR: The Learning Organization.

4 Steen, L. A. (1999). *Assessing Assessment. Assessment Practices in Undergraduate Mathematics*. Washington, DC: Mathematical Association of America, n. 49, pp. 1–6.

5 Allen, M. J. (2004). *Accessing Academic Programs in Higher Education*. Bolton, MA: Anker Publications.

6 Fabry, V. J., Eisenbach, R., Curry, R. R., & Golich, V. L. (1997). Thank You for Asking: Classroom Assessment Techniques and Students' Perceptions of Learning. *Journal on Excellence in College Teaching*, 8(1), pp. 3–21.

7 Angelo, T. & Cross, P. (1993). *Classroom Assessment Techniques, A Handbook for College Teachers.* Second Edition. San Francisco, CA: Jossey-Bass, p. 33.

8 Fabry et.al. (1997).

9 Prepared by the working groups of the Commission on Teaching Standards for School Mathematics of the National Council of Teachers of Mathematics (1995). *Assessment Standards for School Mathematics*. Reston, VA: National Council of Teachers of Mathematics.

10 Wilson, L. D. & Kennedy, P. A. (2003). Classroom Large-Scale Assessment. In Kilpatrick, J., Martin, W. G., and Schifter, D. (Eds.) *A Research Companion to Principles and Standards for School Mathematics*. Reston, VA: National Council of Teachers of Mathematics, p. 64.

11 American Association of Higher Education (AAHE) (2004). *9 Principles of Good Practice for Assessing Student Learning*. Retrieved 3/21/2006 from http://web.archive.org/web/20050305204704/http://www.aahe.org/principl.htm

12 Diamond, R. M. (1998). *Designing and Assessing Courses and Curricula*. San Francisco, CA: Jossey-Bass.

13 Solomon, P. G. (2002). *The Assessment Bridge: Positive Ways to Link Tests to Learning, Standards, and Curriculum Improvement*. Thousand Oaks, CA: Corwin Press, Inc.

Beyond CROSSROADS

Chapter 6

Curriculum and Program Development

Although mathematical truths may be timeless, the content of mathematics programs and courses continues to evolve, often as a result of rapid technological advances. This chapter focuses on the improvement of student learning in mathematics by developing, assessing, and improving mathematics courses and programs. The intent is not to prescribe specific mathematics course content but, rather, to emphasize key components of curricular design and development and encourage periodic review and revision of curricula with the goal of continuous improvement.

Implementation Standard: Curriculum and Program Development

Mathematics departments will develop, implement, assess, and revise courses, course sequences, and programs to help students attain a higher level of quantitative literacy and achieve their academic and career goals.

The Changing Curriculum

Creating a responsive learning environment, assessing student learning, and improving student learning in mathematics requires a dynamic curriculum. There are a number of factors that influence the content and organization of the mathematics curriculum.

- Advances in technology influence both *how* mathematics is taught and *what* mathematics is taught. For example, the square root algorithm and trigonometric tables are rarely used today, whereas spreadsheets and graphing technology are widely used.
- Educational research provides information about how students learn mathematics. These findings should influence how mathematics is taught and how curricula are designed. For example, a well-developed understanding of functions facilitates the transition to advanced mathematical thinking.[1]
- The mathematics needed for successful careers and responsible citizenship continues to change. To be productive, citizens need to be quantitatively literate. For example, individuals are bombarded with information,

The world has gone quantitative: business, geography, criminal justice, history, allied health fields—a full range of disciplines and job tasks tells students why math requirements are not just some abstract school exercise.

U.S. Department of Education, *The Toolbox Revisited: Paths to Degree Completion from High School through College*, February 2006, p. xix.

in their daily lives and in their jobs. To make informed decisions and understand issues, citizens must be able to analyze data, reason with statistics, and understand mathematical models.

- The global economy has changed people's lives and the workplace. The number of unskilled labor positions and jobs that require repetitive processes is decreasing. Conversely, the need for employees who are innovative, can use technology, and are able to think critically is increasing. For example, many processes are now automated, replacing production workers. However, the need for trained technicians who can understand those automations and repair them continues to increase.
- A decrease in the number of students choosing science, technology, engineering, or mathematics as a major, as well as a shortage of instructors in these areas, is affecting the nation's ability to compete in this global economy. In order to find competent employees at lesser salaries, many corporations have established technical support services based abroad.

In response to these factors, faculty need to take the lead in careful, deliberate, informed curricular revision decisions. Research in mathematics education can provide helpful information to guide decision-making in curricular design. The challenge is to design curricula that address the needs of as many academic paths and disciplines as possible. To meet the diverse needs of all students mathematics departments need to collaborate with others (faculty in other disciplines, faculty in four-year institutions, business and industry representatives) to determine appropriate mathematics outcomes. Once the outcomes are established, faculty must access the degree to which students meet those outcomes.

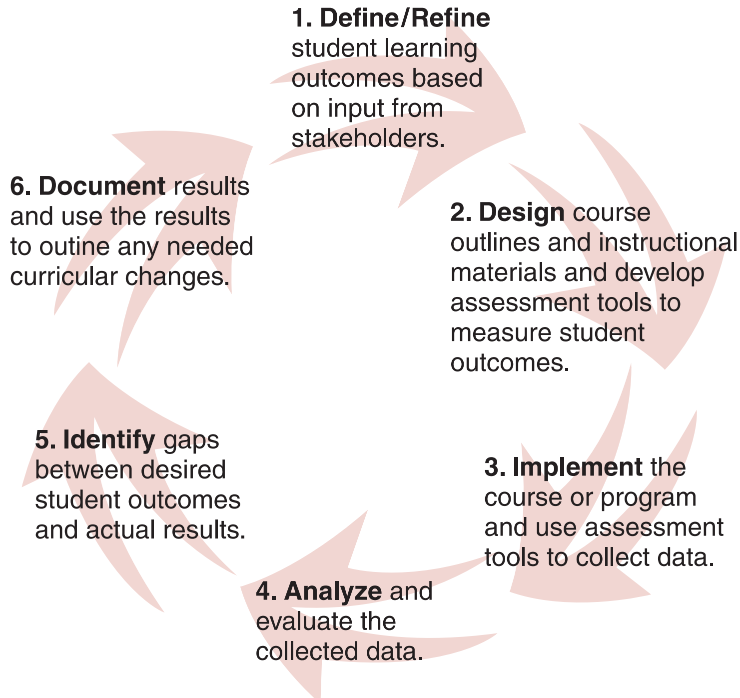
During the last decade, several key curricular issues have stimulated dialogue and educational research. For example, since 1989, when it was suggested that "If it does nothing else, undergraduate mathematics should help students develop function sense...,"[2] considerable research has been conducted on what it means for students to have an understanding of function. Studies report that a well-developed understanding of function correlates closely with success in calculus, as well as facilitating the transition to advanced mathematical thinking.[3] In addition, faculty continue to search for methods to develop a student's understanding of the concept of variable. Students who are able to view variables as representing quantities whose values change dynamically along a continuum have been shown to have ready access to fundamental ideas, such as rate of change and limits, and exhibit higher levels of achievement in mathematics.[4]

There have also been lively discussions about what is the appropriate content of general education mathematics, college algebra, and precalculus. For example, do students need to analyze data and construct appropriate models of real-world phenomena? If so, then curricula need to be designed so that students understand the difference between a constant rate of change (additive, linear) and a constant growth factor (multiplicative, exponential). While learning to distinguish between the two processes, students develop fluency in pattern recognition, data analysis and proportional reasoning.[5] These discussions, within a department, institution, or professional organization are valuable professional development opportunities for all. Faculty and departments are encouraged to continue discussions, consult mathematics education research, and consider educational innovations when designing a curriculum.

The goal and challenge for faculty is to create courses and programs that empower students to become confident and competent problem solvers. Courses should provide opportunities to develop the quantitative skills they will need in their academic work, in society, and in the workplace. The dynamic nature of the curriculum requires a continuous reexamination of how, what, and when mathematics content is taught. The focus of curricular development should be on the mathematics content that is most appropriate for student learning.

The *Beyond Crossroads* Implementation Cycle can be applied to curriculum development, outlining a process for identifying goals, assessing strengths and weaknesses, and implementing curricular change as shown in Figure 3.

Figure 3 The Curriculum Implementation Cycle



Quantitative Literacy

The use of quantitative ideas and language is pervasive in society, news stories, scientific reports, and advertising. Because information is often presented in a variety of representations—words, symbols, tables, or graphs—individuals need to be able to interpret, analyze, and draw conclusions about information presented in these forms. It is important that all citizens have an understanding of the magnitude of numbers, be able to compute and apply percentages, and apply the basic concepts of statistics. In order to be informed consumers, to interpret economic and political trends, or to evaluate health risks of new drugs or treatments, individuals need to understand basic concepts of proportionality, linear and exponential growth, and elementary descriptive and inferential statistics. While entry level jobs may not require significant quantitative reasoning, higher levels of quantitative literacy may be needed to keep open the doors to job advancement.

For these reasons, quantitative literacy should be an outcome of all programs within the college. While most professionals agree on the need for quantitative literacy for all students, there are variations in terminology and definition. Quantitative literacy, quantitative reasoning, numeracy, and mathematics across the curriculum all share many common components. Definitions of quantitative literacy outline student outcomes that range from acquiring specific mathematical skills to gaining mathematical confidence. Quantitative literacy may include the following:

- The ability to apply aspects of mathematics (including arithmetic, measurement, data representation, number sense, variables, geometric shapes, spatial visualization, and chance) to understand, predict, and control routine events in people's lives;[6] the ability to apply arithmetic operations, either alone or sequentially, in many contexts including balancing a checkbook and completing tax forms.[7]

Numeracy [QL] is not the same as mathematics, nor is it an alternative to mathematics. Rather, it is an equal and supporting partner in helping students learn to cope with the quantitative demands of modern society. Whereas mathematics is a well-established discipline, numeracy is necessarily interdisciplinary.

National Council on Education and the Disciplines, *Mathematics and Democracy: The Case for Quantitative Literacy,* 2001, p. 115.

- Statistical reasoning skills and the comfort and confidence to deal with fundamental quantitative problems using critical thinking and problem-solving skills.
- Knowledge of the power and utility of mathematics and how it has shaped civilization.[8]

Acknowledging that more than one definition exists, *Beyond Crossroads* accepts the definition of quantitative literacy presented in the basic principle for quantitative literacy in Chapter 2: an individual's "capacity to identify, understand and engage in mathematics as well as make well-founded mathematical judgments as a constructive, concerned and reflective citizen."[9] Quantitative literacy includes "five different dimensions of numeracy: *practical*, for immediate use in the routine tasks of life; *civic*, to understand major public policy issues; *professional*, to provide skills necessary for employment; *recreational*, to appreciate and understand games, sports, and lotteries; and *cultural*, as part of the tapestry of civilization."[10] While the specific definitions may vary, quantitative literacy is an important general education outcome for every college student. Thus, quantitative literacy outcomes should be woven into every mathematics course and as many other college courses as possible.

Quantitative literacy is more about habits of mind than specific mathematical content. Therefore, the responsibility for developing quantitative literacy, like writing across the curriculum, is shared by the entire college faculty. However, mathematics faculty should lead the quantitative literacy movement by helping to establish a set of outcomes expected of students in each program. There are some outcomes expected of all students.

Students in all college programs will be expected to do the following:

- exhibit perseverance, ability, and confidence to use mathematics to solve problems
- perform mental arithmetic and use proportional reasoning
- estimate and check answers to problems and determine the reasonableness of results
- use geometric concepts and representations in solving problems
- collect, organize, analyze data, and interpret various representations of data, including graphs and tables
- use a variety of problem-solving strategies and exhibit logical thinking
- use basic descriptive statistics
- utilize linear, exponential, and other nonlinear models as appropriate
- communicate findings both in writing and orally using appropriate mathematical language and symbolism with supporting data and graphs
- work effectively with others to solve problems
- demonstrate an understanding and an appreciation of the positive role of mathematics in their lives.

Implementation recommendation: Faculty will integrate quantitative literacy outcomes into all mathematics courses and collaborate with faculty in other disciplines to integrate quantitative literacy into coursework across all disciplines.

Actions to support this recommendation

Faculty and departmental actions:

- determine appropriate quantitative literacy outcomes for each mathematics course and include these outcomes in course outlines
- assume a leadership role in experimenting with new instructional materials to develop quantitative literacy and evaluating and sharing the results
- promote mathematics across the curriculum and positive attitudes by all faculty and students towards mathematics
- initiate collaborations with faculty from other disciplines to promote the integration of quantitative literacy in all programs at the college.
- discuss course content and uses of technology in the mathematical sciences.

Institutional actions:

- promote general education outcomes that include quantitative literacy
- provide opportunities for continuing conversations throughout the college on student outcomes in quantitative literacy
- support faculty from all disciplines in developing and implementing courses that integrate quantitative literacy.

Developmental Mathematics Courses and Programs

Developmental mathematics courses in this document are defined to be courses below the level of the first mathematics course that earns full college credit at the institution. For most two-year colleges, this includes mathematics courses below the level of intermediate or college algebra.

Developmental mathematics courses, to a greater extent than other mathematics courses, serve students with various degrees of success in previous mathematics courses. Appropriate student placement is crucial for developing positive attitudes and maximizing success. The goals of a solid preparation in basic mathematics and swift progress through the curriculum can be compatible. Substantial changes to traditional developmental mathematics curricula, which were derived primarily from high school curricula, need to be implemented. The developmental mathematics program needs to be designed to accomplish agreed-upon goals designed with input from stakeholders, using instructional strategies appropriate for diverse student learning styles and diverse teaching styles. In order to help students who have not previously been successful in mathematics and to implement the recommendations in this report, faculty need to do *more* than teach the same mathematics again.

The desired student outcomes for developmental mathematics courses should be developed in cooperation with the partner disciplines. The content for these courses also should address mathematics anxiety, develop study and workplace skills, promote basic quantitative literacy, and create active problem solvers. The curriculum of developmental mathematics programs should do the following:

- develop mathematical knowledge and skills so students can successfully pursue their career goals, consider other career goals, and function as successful citizens
- develop students' study skills and workplace skills to enable them to be successful in other courses and in their careers
- help students progress through their chosen curriculum as quickly as possible.

Teaching a developmental mathematics course presents unique challenges. Each instructor needs to provide a positive, nurturing experience for each student. Students need to develop time-management skills and study habits. They should engage in activities to help them view mathematics as interrelated concepts of a formal system, not just as unrelated facts to be memorized. While students need skills such

Every course should incorporate activities that will help all students' progress in developing analytical, critical reasoning, problem-solving, and communication skills and acquiring habits of the mind.

Mathematical Association of America, *Undergraduate Programs and Courses in the Mathematical Sciences: CUPM Curriculum Guide 2004,* Executive Summary, p. 1

as solving proportions, they also need to *understand* the concept of proportion and recognize its application in their lives and careers. Students should be able to approach mathematics through contextual, concrete, and abstract situations; apply mathematical skills to solve problems; and be able to transfer their knowledge to new situations. Students should experience multistep problems and be comfortable working in groups and doing collaborative projects. They also should have successful experiences using technology, including calculators, spreadsheets, and other computer software, as a tool to collect, organize, and analyze data, as well as to recognize numerical and graphical patterns.

Topics in developmental mathematics courses should be taught in depth and not as a preview of topics to come in later courses. For many students, any given mathematics course could be their last one. Some algebraic topics, such as factoring, radicals, and operations with rational expressions, should receive less attention, while modeling, communication, and quantitative literacy and reasoning should receive more attention. Topics in algebra, geometry, statistics, problem solving and experience using technology should be integrated throughout developmental courses. However, students should still be expected to perform single digit arithmetic, without the use of a calculator. In addition to the quantitative literacy outcomes and workplace skills expected of all students, there are some global outcomes that are expected of all developmental mathematics students.

Developmental mathematics students will be expected to do the following:

- apply strategies to manage mathematics anxiety
- develop mathematical skills needed to complete other courses successfully
- gain confidence in doing mathematics and solving real-world problems.

Implementation recommendation: Mathematics departments will design and offer developmental mathematics courses that prepare students for the workplace and build mathematical skills needed for informed citizenship and success in a variety of academic and career paths.

Actions to support this recommendation

Faculty actions:

- design courses and classes to produce desired student outcomes in developmental mathematics
- be actively involved in student placement to ensure each student is placed appropriately
- be sensitive to the impact of mathematics anxiety on students and employ strategies to control, manage, and reduce student anxiety
- provide opportunities to develop student confidence in mathematics and problem solving
- demonstrate and encourage multiple problem-solving strategies using appropriate tools from algebra, geometry, and statistics
- seek ways to integrate technology into developmental mathematics courses as a tool to investigate and promote understanding of mathematical concepts.

Departmental actions:

- create developmental mathematics courses and programs for students who only need to refresh their knowledge of basic concepts
- create developmental mathematics courses and programs for students who are taking developmental mathematics for the first time

- collaborate with faculty from other disciplines to ensure the student outcomes for developmental mathematics courses are appropriate.

Institutional actions:

- provide appropriate support services for mathematics students
- provide professional development for mathematics faculty focusing on teaching developmental mathematics.

General Education Mathematics Courses

For many students, a general education course such as logic, statistics, liberal arts mathematics, finite mathematics, or discrete mathematics may be the only mathematics course required for graduation. A general education mathematics course may not be a prerequisite to any subsequent course, but a prelude to the student's experiences as a productive employee and informed, quantitatively literate citizen. As a result, it is critical that general education mathematics courses exemplify the principles and standards of this document. A general education mathematics course may mold students' opinions of mathematics for the rest of their lives. These courses should pay particular attention to influencing positively student attitudes toward mathematics.

The mathematician's patterns, like the painter's or poet's, must be beautiful; the ideas, like the colours or the words, must fit together in a harmonious way.

Godfrey Harold Hardy, *A Mathematician's Apology,* 1941, p. 25.

General education courses should include the important general education outcomes of writing, critical thinking, and quantitative literacy. While the course goals and student outcomes may vary from program to program, all general education courses in mathematics should require students to synthesize, make connections, and use basic mathematical knowledge to solve real-world problems. This synthesis may occur in a variety of mathematical contexts through exploration, discovery, and problem solving. A course may be designed to focus on mathematics as a language used to describe relationships and patterns. Another course might examine how one quantity varies with another (especially in linear, quadratic, and exponential relationships using real data) through a study of functions, geometric patterns, or statistics. Many kinds of investigations, such as using real data, integrating algebraic or geometric structures, or studying statistics, could be used as the core of a general education mathematics course.

In addition to the quantitative literacy outcomes expected of all students, the following are expectations of general education students.

General education mathematics students will be expected to do the following:

- use mathematics and technology to investigate, model, and solve a variety of real-world problems
- use a variety of problem-solving methods
- use mathematics to write and communicate in their discipline
- view mathematics as a language to describe relationships and patterns
- synthesize their mathematical knowledge.

Implementation recommendation: Mathematics departments will design and offer general education mathematics courses for students who take a single college-level mathematics course in their college degree program.

Actions to support this recommendation

Faculty actions:

- collaborate with faculty from other disciplines to determine learning outcomes for general education mathematics courses
- develop and offer mathematics courses to meet general education requirements that model basic principles and standards of this document
- demonstrate the use of technology to find patterns, test conjectures, and discover properties.

Technical and Career Courses and Programs

Mathematics courses in technical programs include topics from arithmetic through calculus. In addition to providing necessary mathematical skills, these mathematics courses should build students' quantitative literacy, develop their ability to think and communicate mathematically, connect mathematics to real-world situations, and develop problem solving skills for working alone, as well as for working in teams.[11]

> **The application of mathematics to technical problems is fundamental to all technical education programs.**
>
> National Science Foundation, *Gaining the Competitive Edge: Critical Issues in Science and Engineering Technician Education,* July 1993, p. 19.

Colleges offering technical and career programs have the responsibility of determining the needs of their local businesses and industries and preparing students to meet these needs. Employers value thinking skills, decision-making, creativity, problem-solving, visualizing and reasoning spatially, and knowing how to learn—as well as personal qualities such as responsibility, self-management, and team skills. Employers need college graduates with reading, writing, speaking, and listening skills, as well as knowledge of basic mathematical concepts.[12] Designing technical courses with mathematics content for specific fields, with flexibility to be used by more than one major, is another challenge. For example, emerging technologies, such as those in the biotechnological fields, tend to require fewer algebraic skills, and more data analysis, modeling, statistics, and discrete mathematics.

Technical mathematics courses and programs should be developed in collaboration with faculty from other disciplines and business and industry representatives to identify and address the mathematics content needs of specific program employers. Content in two-year technical mathematics courses should be selected because of its application to a specific technical field and the needs of specific employers. It should also be at a level equivalent to mathematics courses that transfer to four-year institutions. The direction of curriculum change in technical and career programs is to emphasize workplace skills within the classroom context and to develop flexible thinkers who can work collaboratively to solve new problems. In addition to quantitative literacy outcomes, technical and career students have unique expectations from their respective programs.

Technical and career students will be expected to do the following:

- develop the mathematical concepts necessary to succeed in their chosen program
- use technology to explore and analyze problems
- analyze and model data from their career fields and use these models to solve problems
- be able to use mathematics to communicate orally and in writing
- develop workplace skills including collaborative problem-solving skills.

Implementation recommendation: Technical and career mathematics courses will incorporate workplace skills and address the unique needs of the programs they serve, while providing students with quantitative literacy and an appropriate mathematical background.

Actions to support this recommendation

Faculty actions:

- consult with representatives from business, industry, and technical disciplines and use their input to keep course content relevant
- incorporate workplace skills as an integral part of curricula
- encourage active student learning and the development of team-building skills with term projects, collaborative projects, portfolios, research, field investigations, or internships
- use technology throughout curricula to discover properties, to develop concepts, and to examine multiple perspectives
- give students experience with the technology skills they will use routinely in the workplace.

Departmental/institutional actions:

- provide professional development for faculty that focuses on technical and career mathematics courses and programs
- provide support for faculty in seeking outside funding to support the technology appropriate for the curriculum.

Teacher Preparation Courses and Programs

Many two-year colleges have the preparation of teachers as part of their mission.[13] Many future teachers take most, if not all, of their college-level science and mathematics courses at two-year colleges. Two-year colleges enroll a large proportion of the nation's underrepresented groups and can assume a key role in recruiting minorities to the teaching profession. Positioned between K–12 schools and four-year colleges, two-year colleges can play an important role in promoting collaboration in a variety of issues, such as articulation and transfer, curriculum development, financial support for students, use of technology, program requirements, statewide teacher licensure requirements, and professional assessments.[14] Because of these factors, two-year colleges play a pivotal role in the recruitment and preparation of teachers.

Two-year colleges are essential partners in the mathematical education of teachers. A large number of future teachers begin their postsecondary study in two-year colleges. In particular, elementary teachers take a significant portion of their college mathematics in two-year colleges.

Conference Board of the Mathematical Sciences, *The Mathematical Education of Teachers,* 2001, p. 9

To be effective teachers of mathematics, "Prospective teachers need mathematics courses that develop a deep understanding of the mathematics they will teach."[15] Courses for prospective teachers should develop the habits of mind of mathematical thinkers and demonstrate flexible, interactive teaching styles. They should demonstrate the role of technology to explore ideas and promote concept development. They also should present the fundamental ideas of school mathematics and be taught by mathematicians who have a serious interest in teacher education. All mathematics courses taken by future teachers of mathematics should be standards-based, use strategies that have demonstrated success for students, and incorporate the NCTM's *Principles and Standards for School Mathematics*.[16]

Some of the courses taught at two-year colleges for prospective teachers of mathematics are addressed in the report *The Mathematical Education of Teachers*.[17] This report includes the following recommendations to mathematics departments regarding both the content and delivery of the curriculum.

- Prospective elementary grade teachers will take at least nine semester hours on the fundamental ideas of elementary school mathematics.
- Prospective middle school mathematics teachers will receive training designed specifically for future middle school teachers, which includes at least twenty-one semester hours of mathematics,

including at least twelve semester hours on fundamental ideas of school mathematics appropriate for middle grades teachers.

- Prospective high school teachers of mathematics will complete the equivalent of an undergraduate major in mathematics, including a six-semester hour capstone course connecting their college mathematics courses with the high school mathematics curriculum.

Mathematics courses must emphasize the connections within mathematics, especially connections with the mathematics that is already familiar to the students. The courses also should illustrate the connection between mathematics and other disciplines. Mathematics instruction for future teachers should model informed teaching practices. These practices include setting high expectations for all students, integrating concepts, actively engaging students in the learning process, emphasizing problem solving and reasoning mathematically, and expecting students to reflect on their learning and to communicate mathematically.

Prospective teachers should have intensive courses in effective teaching methods in mathematics.[18] Included in these courses should be a discussion of interactions between the learner and the instructor, among learners, and between learners and the content they are trying to master. Whole group, small group, and individual work should be utilized. Instructors should integrate assessment into instruction and use multiple assessment techniques.

Teachers tend to teach the way they were taught and many school children are concrete learners. Therefore, future teachers—especially future elementary teachers—need preservice experience using manipulatives as a tool to teach mathematics. Manipulatives help develop conceptual understanding and assist students in discovering underlying procedures. Future teachers need experience using the same manipulatives they will encounter in pre-K–12 classrooms. They also need to practice with using the Internet, spreadsheets, mathematics applets, and mathematics software, as tools to teach mathematics.[19]

In the first two years of college, prospective teachers need supervised experiences in classrooms to observe and practice what they are learning in their mathematics courses. These early experiences help students determine whether teaching is the right career choice for them. Fieldwork experiences prior to student teaching afford students the opportunity to begin to become reflective practitioners and evolve in their understanding of educational practices and issues as they analyze, synthesize, and summarize their experiences. Mathematics departments should coordinate with other disciplines to afford prospective teachers the opportunity to participate in early fieldwork experiences.

In addition to providing the first two years of a college-level teacher preparation program, many two-year colleges also offer a variety of other education-related programs. Examples include degree programs in paraprofessional education and early childhood development, professional development workshops and licensure renewal courses for teachers, and teacher licensure programs for professionals holding bachelor's degrees who desire to switch their careers. In some states, colleges formerly considered two-year colleges are now granting bachelor's degrees in high-need disciplines, such as mathematics or science teaching and health care. Recommendations about the mathematical preparation of future teachers also apply to those enrolled in other education-related courses and programs at two-year colleges.
The expectations of future teachers extend beyond the quantitative literacy outcomes and workplace skills outlined earlier.

Students in teacher preparation programs will be expected to do the following:

- develop a deep understanding of the mathematics they will teach and of the teaching practices that result in students' learning of mathematics
- include the manipulatives and technology used in pre-K–12 mathematics classrooms appropriately when designing lesson plans
- communicate mathematical concepts clearly both verbally and in writing
- clarify their career goals through supervised field experiences.

Implementation recommendation: Mathematics departments will create a mathematics program with courses that provide future teachers of mathematics with content knowledge that is beyond and deeper than the mathematics they will be teaching and that exemplifies successful pedagogical methods.

Actions to support this recommendation

Faculty actions:

- model the appropriate use of manipulatives and technology
- use multiple assessment strategies to assess students
- include supervised field experiences for future teachers in some of the mathematics courses
- recruit students from underrepresented groups into the mathematics teaching profession.

Departmental/institutional actions:

- hire faculty with strong mathematics and education backgrounds to teach in teacher-preparation programs
- place teacher preparation as a high priority in the department and have these courses taught by faculty members who have teacher education as one of their primary interests
- provide opportunities for professional development field experiences in collaboration with area four-year institutions and school districts
- work with four-year institutions and state departments of education to develop post-baccalaureate teacher licensure programs.

Mathematics-Intensive Courses and Programs

Mathematics-intensive programs serve students who will major in mathematics or mathematics-dependent fields, including secondary mathematics education, business, computer science, engineering, and science. The demand for a mathematically educated workforce has grown, but the number of students majoring in mathematics has declined. It is essential that students, especially those from underrepresented groups, be provided with a supportive learning environment that encourages their enrollment and retention in mathematics-intensive programs.

A solid understanding of functions and proof is a critical component of students' mathematical foundation. Each student needs to develop an ability to work at an abstract level. Students should use rich applications from various disciplines to deepen their appreciation and understanding of the power of mathematics to model real-world phenomena. These applications can be used to promote active and collaborative learning, practice sustained effort, examine a variety of problem-solving techniques, and communicate mathematically by giving written or oral reports. Mathematics-intensive programs should increase students' understanding of the mathematics topics studied in calculus. The expectations of mathematics-intensive students extend well beyond the quantitative literacy outcomes expected of all students.

In fall 1972, President Nixon announced that the rate of increase of inflation was decreasing. This was the first time a sitting president used the third derivative to advance his case for reelection.

Hugo Rossi,
Notices of the American Mathematical Society,
October 1996, p. 1108.

Mathematics-intensive students will be expected to do the following:

- develop an appreciation of mathematics as a whole and of the historical development of mathematics
- develop a solid understanding of functions from multiple perspectives

- be able to use numerical, graphical, symbolic, and verbal representations to solve problems and communicate with others
- use technology as a tool for exploring mathematical concepts
- use a variety of mathematical models including curve fitting
- develop an ability to work with mathematical abstractions, analyze mathematical relationships, make plausible conjectures, projections, and develop proofs
- develop an understanding of concepts and skills needed for future mathematics courses or courses in related disciplines.

Implementation recommendation: A mathematics-intensive program will promote students' appreciation and zeal for mathematics and develop the skills, concepts, and problem-solving and communication strategies necessary for success in their courses and careers.

Actions to support this recommendation

Faculty actions:

- create courses and activities to promote students' ability to use multiple approaches or representations to examine mathematical concepts so that students develop a better understanding of connections among topics and improve their ability to work abstractly
- require group projects, group discussions, and explorations to promote the ability of students to use mathematical notation and terminology
- expect students to communicate orally and with written reports
- use technology to promote student discovery, develop concepts, examine multiple perspectives, and give students experience with the technology skills that they will use in their careers
- work with instructors in other disciplines to develop learning communities that pair a mathematics class with a class in another department
- provide guest speakers from scientific fields for student mathematics clubs or for selected classes
- encourage student participation in professional organizations through activities such as the AMATYC Student Math League, MAA student chapters, and Mu Alpha Theta mathematics organizations
- recruit students, including those from underrepresented groups, into mathematics-intensive programs and careers.

Conclusion

Mathematics courses and programs in the first two years of college need to develop students' quantitative and workplace skills and actively engage them in the mathematics they will encounter outside the classroom. Faculty may need to teach content that is *different* from what they were taught, teach *more* than they were taught, and teach *differently* than the way they were taught. Students should understand some of the big ideas of mathematics through a curriculum, a variety of problem-solving strategies, and significant projects that examine selected topics in depth. Students should have opportunities to demonstrate their mathematical knowledge, as well as their creativity. When designing mathematics curricula, faculty and departments should consider the needs of each student, as well as the mathematical requirements of other courses and the workplace, and employ multiple approaches in instructional strategies. In addition, each course and program should be reviewed periodically for continuous improvement. The entire mathematics community needs to embrace continuous improvement in curriculum and program development as the process that can increase student learning.

HIGHLIGHTS Implementing the Standards for Curriculum and Program Development

Curriculum and Program Development

Mathematics departments will develop, implement, assess, and revise courses, course sequences, and programs to help students attain a higher level of quantitative literacy and achieve their academic and career goals.

At a standards-based institution, the *faculty*

- ☑ integrate quantitative literacy outcomes into all mathematics courses and into coursework across all disciplines.
- ☑ collaborate with partner disciplines and business and industry to establish desired student outcomes.
- ☑ create courses and programs based on desired student outcomes.
- ☑ lead the periodic review and revision of an up-to-date curriculum.
- ☑ use technology throughout the curriculum to examine concepts from multiple perspectives, and to explore, discover, and develop technology skills needed in the workplace.
- ☑ teach topics in developmental courses in depth (rather than a quick review of high school topics) and promote classroom and workplace skills.
- ☑ emphasize for general education students the connections among some of the big ideas of mathematics.
- ☑ collect, analyze, and model data from realistic applications in technical, career, and other mathematics courses.
- ☑ promote the zeal for mathematics in mathematics-intensive students by examining big ideas in depth and using multiple approaches or representations to reveal the connections among these ideas and with topics outside of mathematics.
- ☑ expect all students to use the language and symbolism of mathematics to communicate effectively with other students.

At a standards-based institution, the *mathematics department* and the *institution*

- ☑ promote quantitative literacy outcomes across the curriculum and in general education courses.
- ☑ provide the facilities, support, and professional development for faculty to create, revise, and teach the curriculum.
- ☑ place teacher preparation as a high priority in the department and reflect this priority in hiring, class assignments, and professional development.
- ☑ continually review and revise curricula with input from two-year and four-year college faculty to maximize course transferability and access to additional educational opportunities.
- ☑ support actions to increase the number of students choosing mathematics-intensive or teacher preparation programs.

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1 Tall, D. O. (1992). The Transition to Advanced Mathematical Thinking: Functions, Limits, Infinity, and Proof. In D. Grouws (Ed.), *Handbook of Research on Mathematics Teaching and Learning*. New York, NY: Macmillan, pp. 495–511.

Carlson, M. (1998). A Cross-Sectional Investigation of the Development of the Function Concept. *Research in Collegiate Mathematics Education III. CBMS Issues in Mathematics Education*. Providence, RI: American Mathematics Society, v. 7, pp. 115–162.

2 National Research Council. (1989). *Everybody Counts: A Report to the Nation on the Future of Mathematics Education*. Washington, DC: National Academy Press, p. 51.

3 Tall (1992); Carlson (1998); Other pedagogical studies about functions:

Use of the input-output machine metaphor: Davis, G. & McGowen, M. (2002). Function Machines and Flexible Algebraic Thought. In A. D. Cockburn & E. Nardi (Eds.), *Proceedings of the 26th International Conference for the Psychology of Mathematics Education*. University of East Anglia, Norwich, U.K.

Covariational reasoning: Carlson, M., Jacobs, S., Coe, T., Hsu, E. & Larsen, S. (2002). Applying Covariational Reasoning While Modeling Dynamic Events: A Framework and a Study. *Journal for Research in Mathematics Education*, Reston, VA: National Council of Teachers of Mathematics, 33(5), pp. 352–378.

Dynamic view of function: Thompson, P. W. (1994). Students, Functions, and the Undergraduate Curriculum in Dubinsky, E., Schoenfeld, A. H., & Kaput, J. (Eds) (1998). *Research in Collegiate Mathematics Education I, CBMS Issues in Mathematics Education*. Providence, RI: American Mathematics Society, v. 4, pp. 21–44.

Establish contexts for modeling: Kaput, J. (1994). Democratizing Access to Calculus: New Routes to Old Roots. In A. H. Schoenfeld (Ed.) *Mathematics and Cognitive Science*. Washington, DC: Mathematical Association of America, pp. 77–156.

Student interaction with visual aspects of function graphs: Monk, S., & Nemirovsky, R. (1994). The Case of Dan: Student Construction of a Functional Situation Through Visual Attributes in Dubinsky, E., Schoenfeld, A. H., & Kaput, J. (Eds.) (1998). *Research in Collegiate Mathematics Education I, CBMS Issues in Mathematics Education*. Providence, RI: American Mathematics Society, v. 4, pp. 139–168.

4 Ursini, S., & Trigueros, M. (1997). Understanding of Different Uses of Variable: A Study with Starting College Students. In Pehkonen, E. (Ed.). *Proceedings of the 21st International Conference for the Psychology of Mathematics Education*. Lahti, Finland, v. 4, pp. 254–261.

Jacobs, S. (2002). *Advanced Placement BC Calculus Students' Ways of Thinking about Variable*. Unpublished doctoral dissertation, Arizona State University, Tempe, AZ.

5 Confrey, J. & Smith, E. (1995). Splitting, Covariation, and Their Role in the Development of Exponential Functions. *Journal for Research in Mathematics Education*. Reston, VA: National Council of Teachers of Mathematics, 26(1), pp. 66–86.

6 QL. *Different Views on Quantitative Literacy*. Retrieved 1/22/2006 from http://www.stolaf.edu/other/extend/Numeracy/defns.html, p. 1.

7 National Council on Education and the Disciplines (NCED). (2001). Steen, L. A. (Ed.). *Mathematics and Democracy. The Case for Quantitative Literacy*. Washington, DC: The National Council on Education and the Disciplines, p. 7.

8 SIAM News. (April 2002). *Quantitative Literacy and SIAM*. Retrieved 1/22/2006 from http:// www-math.cudenver.edu/~wbriggs/qr/siam_news.html, p. 1.

9 Organization for Economic Cooperation and Development, UNESCO Institute for Statistics (2003). *Literacy Skills for the World of Tomorrow—Further Results from PISA 2000*. Paris, France: Organization for Economic Cooperation and Development Publishing, p. 20.

10 Steen, L. A. (1997). *Why Numbers Count: Quantitative Literacy for Tomorrow's America*. New York, NY: The College Entrance Examination Board, p. xxii.

11 Mathematical Association of America Subcommittee on Curriculum Reform across the First Two Years (CRAFTY). (2004). *The Curriculum Foundations Project*. Washington, DC: Mathematical Association of America.

12 U.S. Department of Labor. Secretary's Commission on Achieving Necessary Skills (SCANS). (1991). *What Work Requires of Education*. Washington, DC: U.S. Department of Labor. *Skills and Tasks for Jobs: A SCANS Report for America 2000*. (1999). Washington, DC: U.S. Department of Labor.

13 Report of the NSF Workshop (December 1999). *Investing in Tomorrow's Teachers: The Integral Role of the Two-Year College in the Science and Mathematics Preparation of Teachers*. Arlington, VA: The National Science Foundation.

14 Example: Praxis Test Series. Retrieved 3/23/2006 from http://www.ets.org/portal/site/ets/menuitem.435c0b5cc7bd0ae7015d9510c3921509/?vgnextoid=48c05ee3d74f4010VgnVCM10000022f95190RCRD.

15 Conference Board of the Mathematical Sciences (CBMS). (2001). *The Mathematical Education of Teachers, Part I*. Washington, DC: Mathematical Association of America, p. 7.

16 National Council of Teachers of Mathematics (NCTM). (2000). *Principles and Standards for School Mathematics*. Reston, VA: National Council of Teachers of Mathematics.

17 CBMS (2001).

18 U.S. Department of Education. The National Commission on Mathematics and Science Teaching in the 21st Century. (2000). *Before It's Too Late*. Washington, DC: U.S. Department of Education.

19 NCTM (2000–2006) *Illuminations*. Reston, VA: National Council of Teachers of Mathematics. Retrieved 3/23/2006 from www.illuminations.nctm.org.

Chapter 7

Instruction

Understanding how students learn mathematics and knowing which instructional methods are likely to be successful should inform instructional practice. Effective mathematics instruction requires a variety of resources, materials, technology, and delivery formats that take into account students' different learning styles and instructors' different teaching styles. Every teaching activity should promote active learning and be guided by informed decision-making.

Implementation Standard: Instruction

Mathematics faculty will use a variety of teaching strategies that reflect the results of research to enhance student learning.

Teaching Styles

"Teaching style" refers to an instructor's content-independent, persistent qualities, attitudes, and traits. It is directly linked to the instructor's educational philosophy and a subset of the instructor's life philosophy.[1] Teaching styles may be student-centered or teacher-centered or any combination of the two. In a student-centered style, the instructor expects the student to take responsibility for learning and the student trusts the instructor will help. Student-centered models involving a constructivist approach and active learning have been shown to be successful for students with non-traditional learning styles.[2] In a teacher-centered style, the instructor organizes the transmission of content knowledge to the learner, acts as a performance assessor, and provides opportunities for the learner to practice independently. Understanding where an instructor fits in the continuum of styles and using that information to make informed decisions about instruction is one of the steps in becoming a professional educator, rather than simply a teacher. Regardless of which teaching style is preferred, using multiple instructional strategies is recommended to address as many individual learning styles (see Chapter 4).

Table 6 Student-Centered and Teacher-Centered Teaching Styles[3]

Student-Centered	Teacher-Centered
Knowledge is constructed by students gathering and synthesizing information.	Knowledge is transmitted from instructor to students.
Emphasis is on using knowledge to address problems students will experience outside of the classroom.	Emphasis is on the acquisition of knowledge, sometimes outside the context in which it will be used.
The instructor's role is coach, facilitator, and mentor; students and instructor assess learning performance together.	The instructor's role is primarily as an information-giver and performance assessor.
Assessment is used to diagnose learning problems, in addition to evaluating learning outcomes.	Assessment is used to evaluate learning outcomes.
Teaching and assessment of learning are intertwined.	Teaching and assessment of learning are separate.
The learning culture is cooperative and collaborative.	The learning culture is competitive and individualistic.
Instructors and students learn together.	Students are viewed as learners.

The diverse needs of students in any mathematics course call for diverse approaches to instruction. While knowledge of content is essential in teaching any discipline, effective teaching is the result of integrating content and pedagogical knowledge.[4] Effective instructors venture outside their comfort zone, take risks, grow in their teaching, and model the behavior they expect from their students.[5] The instructional methods chosen can encourage or inhibit students' ability to demonstrate their mathematical knowledge and take ownership for creating and understanding mathematical ideas.[6] Certain teacher practices such as providing students with frequent opportunities to experience problem solving in a variety of contexts, allowing adequate time for solving problems, devoting time specifically for planning, monitoring, and reflecting on progress and the answer, have been shown to help students improve problem-solving skills. "A variety of instructional strategies are necessary to respond to as many different learning and teaching styles as possible."[7]

…students who can think mathematically and reason through problems are better able to face the challenges of careers in other disciplines—including those in nonscientific areas.

Mathematical Association of America's Curriculum Renewal Across the First Two Years Subcommittee (CRAFTY), *The Curriculum Foundations Project, Voices of Partner Disciplines,* 2004, p. 1

Mathematics instruction may be improved through sensitivity to learning styles. However, matching teaching style to learning style has limitations and does not necessarily guarantee greater student achievement.[8] One meta-study involving children concluded that there was little effect on learning when a teaching strategy was chosen specifically to respond to a student's particular learning style.[9] In other research involving college students, matching students' learning styles with a variety of instructional strategies showed some success, particularly with new students and poorly prepared students, where most attrition occurs.[10] To become more aware of the kinds of methods an instructor is selecting, the instructor can code lesson plans by placing a V next to visual activities, an R next to reflective activities, etc. "If the coding system is used on a regular basis, it becomes very natural to think in terms of being inclusive, or providing the setting and the activities by which all learners can find some portion of the class that particularly appeals to them."[11] In selecting instructional strategies, the goal should be to find the best instructional strategy for the *particular* content being taught[12] and for the particular level of cognitive development of the students in the class.

Implementation recommendation: Faculty will recognize their own teaching style(s), reflect on the implications of their style on their students' learning styles, and use that knowledge and other research to make informed decisions about the selection of multiple instructional designs and their classroom management.

Actions to support this recommendation

Faculty actions:

- respect diverse talents and ways of learning and teaching
- design and use a variety of classroom activities, assignments, and assessments to address multiple learning styles
- encourage student-faculty contact
- provide students with prompt feedback
- be attentive, expressive, enthusiastic, clear, and organized
- present multiple representations of mathematical concepts.

Active Student Learning

Many college students today have grown up with technology and as a result, they process information differently than students who did not. This characteristic has implications for mathematics classes, in general, not just courses using calculators or computers. When planning classroom activities, instructors need to acknowledge that their students[13]

- are intuitive visual communicators
- learn better by making observations or through discovery; they like doing things, not just thinking or talking about things
- can shift their attention rapidly from one task to another
- are able to respond quickly and expect rapid responses in return
- prefer to work in teams.

For today's students, learning is participatory—*knowing* depends on practice and participation.

New findings in the science of learning suggest that learning requires some sort of cognitive restructuring of material by the learner. "Transfer can be improved by helping students become more aware of themselves as learners who actively monitor their learning strategies and resources and assess their readiness for particular tests and performances."[14] Therefore, instructors need to draw out and work with pre-existing understandings students bring with them.

Collaborative and cooperative learning are examples of strategies to encourage active student learning. Collaborative learning is defined strictly as an unstructured process in which participants define problems, develop procedures, and produce socially constructed knowledge. Collaborative learning develops social skills, promotes student-student and student-faculty interactions and builds self-esteem in students as it develops mathematics skills and knowledge.[15] A shy student, when placed in a small group, still benefits from collaborative learning by observing strategies used by his/her peers. One of the most effective means of restructuring is by explaining the material to someone else.[16] Cooperative learning is a structured, systematic instructional strategy in which groups work toward a common goal.[17] In practice, most group learning activities are a mix of cooperative and collaborative learning. The goal of both collaborative and cooperative learning is for students to be actively involved in learning activities that lead to a rich understanding of the course content.

If students in mathematics classes are to learn mathematics with understanding…then it is important to examine the roles of the teacher and the knowledge that underlies the teacher's enactments of those roles.

National Research Council, *How People Learn: Brain, Mind, Experience, and School.* 1999, p. 152

Interactive lecturing, question posing, and inquiry-based strategies have also been shown to increase students' active learning, interest, and curiosity.[18] Designing questions and problems that are challenging yet within student's cognitive development and giving students time to respond are critically important for these instructional methods to be effective. Discovery-based learning utilizing carefully designed questions and activities can lead students to connect new knowledge to previous knowledge. Additional questions may be necessary to facilitate knowledge construction for some students. Allowing a learner to grapple with the logic behind a rule and then apply it may be frustrating, but develops understanding of concepts.[19]

The appropriate use of writing in mathematics courses is another strategy that can play an important role in the process of internalizing mathematical procedures, understanding the relationships of mathematical concepts, and synthesizing different mathematical components into a coherent schema.[20] Writing "is an essential activity to create order from chaos, sense from nonsense, meaning from confusion: as such it is the heart of creative learning in both the arts and sciences."[21]

Active learning occurs in many formats such as collaborative learning, discovery-based learning, interactive lecturing and question posing, and writing. Whichever format is chosen, the goal of the activity should be to enhance conceptual understanding.

Table 7 Instructional Strategies That Promote Active Learning

	Strategies for Faculty
Collaborative/Cooperative Learning	• Provide pairs or small groups of students the opportunity to play a mathematical game, solve a textbook problem, explain, engage in a research project, interpret a graph to another student, review another student's work, discuss the correct solution, review and critique a video. • Assign Internet group projects (in or out of class), technology activities or activities where measurements are taken and analyzed. • Facilitate informal study groups outside of class. • Ask students to work in pairs on a homework problem before class begins. • Provide a location for group review for an upcoming test.
Discovery-based Learning	• Use student answers to guide classroom discussion. • Present examples that lead to patterns, which form the basis of mathematical rules. • Ask students to discover concepts and patterns.
Interactive Lecturing and Question-Posing	• Ask students to raise their hand and select the third or fourth hand for a response. • Ask all students to solve a problem and then compare their processes and/or answer with a student nearby. • Respond to any answer given (right or wrong) and discuss the logical ramifications of trying to solve the problem using alternative methods. • Ask questions to guide students to solutions to problems. • Engage students in activities that lead them to develop conceptual understandings. • Ask students to write questions that require explanations.
Writing	• Ask students to write to an absent student explaining the most important mathematics concept from the day's lesson. • Ask students at the beginning of the class to write about what they learned from doing homework. • Encourage students to take notes and/or keep track of important mathematics ideas and problem statements by writing a checklist on the board or disseminating a fact sheet. • Ask members of a group to describe how to perform mathematical processes integral to the mathematical concept. • Include a writing component in homework, student projects, investigations, and explorations of mathematical concepts.

Implementation recommendation: Faculty will design and implement instructional activities that actively engage students in the learning of mathematics.

Actions to support this recommendation

Faculty actions:

- design and implement multiple instructional approaches that promote active student participation in the learning process
- formulate activities that require students to memorize, comprehend, apply, analyze, and synthesize mathematical concepts
- allow discovery-based and thought-provoking questions and activities to guide classroom discussions
- provide opportunities for and encourage students to think, reflect, discuss, and write about mathematical ideas and concepts.

Departmental/institutional actions:

- provide faculty with appropriate facilities and technology to design and implement instructional strategies that engage students actively
- provide faculty with professional development opportunities to improve their instructional strategies.

Teaching with Technology

Advances in hardware and software technology have had a tremendous impact on our lives. The infusion of technology into education presents interesting opportunities for teaching and learning, especially in mathematics. Technology changes not only *how* mathematics is taught, but also *when* and *what* mathematics is taught.

The definition and use of technology in mathematics education is constantly evolving. Technology may refer to the use of graphing calculators, student response systems, online laboratories, simulations and visualizations, mathematical software, spreadsheets, multimedia, computers or the Internet, and other innovations yet to be discovered. Technology can be used to learn mathematics, to do mathematics, and to communicate mathematical information and ideas. The Internet hosts a wealth of mathematical materials that are easily accessible through the use of search engines, creating additional avenues to enhance teaching and facilitate learning. Outside of class, students and faculty can pose problems and offer solutions through e-mail, chat rooms, or websites. Technology provides opportunities for educators to develop and nurture learning communities, embrace collaboration, provide community-based learning, and address diverse learning styles of students and teaching styles of teachers.[22]

Faculty using technology should answer the question: How can the use of technology facilitate learning that is durable, has substance, is engaging to students, and provides mathematical insights through a high level of understanding of the mathematics being taught? They should expose students to a variety of technology options appropriate for solving different types of problems and guide them in making appropriate choices about the use of technology for problem solving.

Technology enhances the learning of mathematics, when used appropriately as a tool, to achieve the following:

- explore new concepts and discover patterns
- examine, organize, analyze, and visualize real-world data
- develop understandings of mathematical ideas

- make connections among and between mathematical ideas
- provide a visualization of mathematical models
- provide symbolic, graphical, and/or numerical evidence to support or dispel student-formulated conjectures.[23]

In addition, technology helps students document the validity of their mathematical/critical thinking process, facilitating and enriching the learning processes and the development of problem-solving skills.

The use of technology should be guided by consideration of what mathematics is to be learned, the ways students might learn it, the research related to successful practices, and the standards and recommendations recommended by professional organizations in education. Technology can be used by mathematics educators to enhance conceptual understanding through a comparison of verbal, numerical, symbolic, and graphical representations of the same problem. Students can use technology to search for patterns in data, while allowing the technology to perform routine and repeated calculations. The use of technology should not be used as a substitute for an understanding of and mastery of basic mathematical skills. Technology should be used to enhance conceptual understanding, while simultaneously improving performance in basic skills.

We are educating a generation of "natives," who have grown up using sophisticated communications and information technologies. This fact alone has helped to transform the way technology is being used and integrated into classroom instruction.

National Science Foundation, *America's Pressing Challenge—Building a Stronger Foundation,* 2006, p. 6

The integration of appropriately used technology can enhance student understanding of mathematics through pattern recognition, connections, and dynamic visualizations. Electronic teaching activities can attract attention to the mathematics to be learned and promote the use of multiple methods. Learning can be enhanced with electronic questioning that engages students with technology in small groups and facilitates skills development through guided-discovery exercise sets. Using electronic devices for communication, all students can answer mathematics questions posed in class and instructors can have an instantaneous record of the answers given by each student. This immediate understanding of what students know, and don't know, can direct the action of the instructor in the teaching session.[24] This cyclic assessment of student learning is an illustration of the *Beyond Crossroads* Implementation Cycle in action. Faculty establish goals for the use of technology in the classroom, establish assessments to measure the activity, and collect and analyze the data to revise and improve the activity.

Students will be expected to use technology to do the following:

- enhance their understanding of mathematics
- discover mathematical concepts and patterns
- perform mathematical tasks
- visualize different representations of the same mathematical concept
- formulate and test conjectures about mathematical concepts and procedural rules
- communicate mathematical information and ideas.

Implementation recommendation: Faculty will integrate technology appropriately into their teaching to enhance students' understanding of mathematical concepts and skills.

Actions to support this recommendation

Faculty actions:

- integrate technology into their teaching of mathematics
- use technology tools for assessment that are aligned with instruction
- align technology platforms with those familiar to students, required for future courses, and/or necessary in their future careers.

Departmental/institutional actions:

- provide technology with options for interactivity between students and faculty supporting classroom activities and student learning of mathematics
- provide technology for students to learn and faculty to teach mathematics courses.

Distance Learning

Distance learning, or learning at a distance, involves alternative delivery formats where the instructor and the student are separated either by time, location, or both. Distance learning courses may be delivered synchronously (in real-time), asynchronously (in delayed time), or a combination of both (hybrid) through written correspondence, text, graphics, various digital media, interactive television, videoconferencing, or online. Instruction can involve a variety of distance learning delivery formats:

- Web-displayed, the traditional classroom with the syllabus and some readings and assignments online
- Web-enhanced, the traditional classroom with some lectures and interactions online
- hybrid/blended, with both the Web-displayed and Web-enhanced components and increased online content and interaction
- completely online.

Faculty across the nation have developed and implemented distance learning courses and programs in an effort to provide alternative delivery formats, flexibility in course scheduling, and accessibility for students. Yearly reports from the National Center for Educational Statistics show an increase in the number of courses offering some degree of technology-based delivery.[25] Education at a distance allows students with work and personal commitments to take courses at their convenience and participate in a dynamic and interactive process for student learning.[26]

> **Although colleges and universities have been offering online instruction since the early 1980s, the World Wide Web has provided them with new means of reaching out not only to their traditional service areas but well beyond…**
>
> M. Simonson, S. Smaldina, M. Albright, and S. Zvacek, *Teaching and Teaching at a Distance*, 2003, p. 233.

The extraordinary growth of technology-mediated mathematics courses in higher education has created new challenges. The pressure to expand the number of courses taught at a distance must be balanced with sound strategies for assessment to assure quality. Students and faculty face new challenges with regard to communicating mathematics through these different media. Faculty face new challenges as they attempt to reach the same goals for problem solving, critical thinking, collaboration, communication, content, and assessment as they have in a seated class. These challenges require that professional organizations provide faculty with opportunities to share experiences and learn from one another through conferences, workshops, and by participating in electronic discussions sponsored by the organizations.

Mathematics distance education programs must be carefully planned so they effectively deliver distance education and achieve the same goals and objectives and student learning outcomes as the same course taught in a more traditional classroom. Developing interactive distance learning tools to promote active learning in mathematics requires appropriate infrastructure, professional development, and resources. Policy and accrediting agencies have outlined benchmarks and factors such as institutional context and commitment, curriculum and course development, teaching and learning, faculty support, student support, and evaluation and assessment to ensure excellence in distance education and reflect quality.[27]

The capacity of the college media and computing services to support a distance learning program is an important consideration. Access and equity must be assured for all students. Student outcomes, expectations, and responsibilities must be clearly outlined and communicated. Orientation and advising, specifically related to distance learning, must be provided to students. Students must be aware of the personal attributes required to be successful in courses that have an alternative delivery format. Faculty and departments should develop strategies that help students make wise decisions about which delivery format is most likely to result in success. In addition, students must have easy access to the hardware and software required in the course. Faculty should consider requiring students to use the platform on which the course will be delivered as a prerequisite to registration. Essential components of a distance learning course include the following:[28]

- a documented technology plan that is in place and operational
- minimum standards that guide course development, design, and delivery based on learning outcomes, not on the availability of existing technology
- students actively engaged in analysis, synthesis, and evaluation of the course
- periodic reviews of learning outcomes and instructional materials
- necessary mathematical software and hardware available to all students enrolled in distance learning mathematics courses
- focus on goals, not on delivery format
- models of course development, instruction, and assessment that address sustainability and course enhancement beyond the involvement of the original developer(s).

The Implementation Cycle of *Beyond Crossroads* can be helpful in defining goals and objectives, assessing effectiveness, and making improvements in distance learning courses on a continuing basis.

Students will be expected to do the following:

- engage in distance learning orientation and advising for distance learning mathematics courses
- interact with faculty and students in the distance learning mathematics courses in which they are enrolled
- assume responsibility for their learning in an alternative delivery format.

Implementation recommendation: Faculty will utilize effective instructional design practices when developing and implementing distance learning courses in mathematics.

Actions to support this recommendation

Faculty actions:

- select technology that is accessible to students enrolled in their distance learning mathematics course
- advise students on the expectations of their distance learning mathematics course and orient them to the distance learning environment of their course
- provide students with course information outlining course objectives, concepts, ideas, and learning outcomes for their distance learning mathematics course
- engage in ongoing professional development to enhance their mathematics course presentation and support their teaching practice in the distance learning environment

- assure that learning outcomes in mathematics distance learning sections are consistent with those of similar mathematics courses taught in classrooms.

Departmental/institutional actions:

- provide and adequately maintain the infrastructure and resources to support the development and teaching of distance learning courses in mathematics
- codify distance learning policies and procedures for mathematics students, faculty, and support staff and develop appropriate instruments for student and peer evaluation of distance learning mathematics courses
- provide distance learning orientation, advising, hands-on training, and support services to students registered for or intending to register for distance learning mathematics courses
- provide professional development to support mathematics faculty developing and teaching distance learning courses
- establish a model for distance learning course development, instruction, and assessment that will be sustainable after the tenure of the developer(s).

Conclusion

The teaching of mathematics should be guided by research in learning theory, should incorporate active student participation, and utilize multiple instructional strategies to maximize student success in mathematics. The use of technology in a mathematics classroom should help students become active learners and develop a deep understanding and profound appreciation of mathematics. Instructional strategies and delivery formats should be designed to respond to different learning and teaching styles, to use technology appropriately to enhance learning, to provide opportunities of access and academic success for all students, and to facilitate avenues for collaboration and cooperation.

Beyond CROSSROADS **HIGHLIGHTS** Implementing the Standard for Instruction

Instruction

Mathematics faculty will use a variety of teaching strategies that reflect the results of research to enhance student learning.

At a standards-based institution, the *faculty*

- ☑ use multiple instructional strategies that encourage active student learning and address different learning and teaching styles.
- ☑ actively manage the learning environment.
- ☑ integrate technology as a tool to help students discover and understand key mathematical concepts.
- ☑ align technology tools for assessment with instruction.

At a standards-based institution, the *mathematics department* and the *institution*

- ☑ provide faculty with the resources and training they need to select, develop and refine curriculum materials and instructional activities.
- ☑ provide the necessary facilities, technology, student services, and training to support understanding, development, and implementation of multiple instructional strategies to address various learning and teaching styles.

1 Conti, G. J. (1998). Identifying Your Teaching Style. In M. Galbraith (Ed.), *Adult Learning Methods: A Guide for Effective Instruction* (2nd Ed.). Malabar, FL: Krieger, pp. 73–84.

Galbraith, M. W. (1998). Becoming an Effective Teacher of Adults. In M. Galbraith (Ed.), *Adult Learning Methods: A Guide for Effective Instruction* (2nd Ed.). Malabar, FL: Krieger, pp. 3–19.

2 Springer, L., Stanne, M. E. & Donovan, S. S. (1999). Effects of Small-Group Learning on Undergraduates in Science, Mathematics, Engineering, and Technology: A Meta-Analysis. *Review of Educational Research*, 69 (1), pp. 21–51.

3 Barr, R. B. & Tagg, J. (1995). From Teaching to Learning—A New Paradigm for Undergraduate Education. *Change*. 27(6).

Bonstingl, J. J. (1992). *Schools of Quality*. Alexandria, VA: Association for Supervision and Curriculum Development.

Boyatzis, R. E., Cowen, S. S., & Kolb, D. A. (1995). *Innovation in Professional Education: Steps on a Journey from Teaching to Learning*. San Francisco, CA: Jossey-Bass.

Duffy, D. K., & Jones, J. W. (1995). *Teaching within the Rhythms of the Semester*. San Francisco, CA: Jossey-Bass.

Kleinsasser, A. M. (1995). Assessment Culture and National Testing. *Clearing House*. 68(4).

4 Conti (1998); Galbraith (1998).

5 Apps, J. W. (1991). *Mastering the Teaching of Adults*. Malabar, FL: Krieger.

6 Schoenfeld, A. H. (1992). *Learning to Think Mathematically: Problem Solving, Metacognition, and Sense Making in Mathematics*. In D. A. Grouws (Ed.), *Handbook of Research on Mathematics Teaching and Learning*. New York, NY: Macmillan, pp. 334–370.

Schoenfeld, A. H. (1998). Reflections on a Course in Mathematics Problem Solving. *Research in Collegiate Mathematics III*. *CBMS Issues in Mathematics Education*, Providence, RI: American Mathematical Society, Vol. 7, pp. 81–113.

7 Galbraith (1998).

8 Brown, B. L. (2003). Teaching Style vs. Learning Style. *Myths and Realities*. No. 26. Clearinghouse on Adult, Career, and Vocational Education. Columbus, OH: The Ohio State University.

9 Kavale, K. A. & Forness, S. R. (1987). Substance over Style: Assessing the Efficacy of Modality Testing and Teaching. *Exceptional Children*, 54(3), pp. 228–239.

10 Claxton, C. S. & Murrell, P. H. (1987). Learning Styles: Implications for Improving Educational Practices. *ASHE-ERIC Higher Education Report*. *No. 4.* Washington, DC: Association for the Study of Higher Education.

Dwyer, K. K. (1998). Communication Apprehension and Learning Style Preference: Correlations and Implications for Teaching. *Communication Education*, 47(2).

Midkiff, R. B. & Thomasson, R. D. (1993). *A Practical Approach to Using Learning Styles in Math Instruction*. Springfield, IL: Charles C. Thomas.

Shaughnessy, M. F. (1998). An Interview with Rita Dunn about Learning Styles. *Clearing House*, 71 (3), pp. 141–45. ERIC document EJ566159.

Sims, R. R. & Sims, S. J. (1995). *The Importance of Learning Styles: Understanding the Implications for Learning, Course Design, and Education*. Contributions to the Study of Education, No. 64. Westport, CT: Greenwood Press.

11 Zhenhui, R (2001). Matching Teaching Styles with Learning Styles in East Asians Contexts. *The Internet TESL Journal*. Retrieved 3/9/2006 from http://iteslj.org/Techniques/Zhenhui-TeachingStyles.html.

12 Willingham, D. T. (2005). Do Visual, Auditory, and Kinesthetic Learners Need Visual, Auditory, and Kinesthetic Instruction? *American Educator*, Summer 2005, American Federation of Teachers. Retrieved 2/13/2006 from www.aft.org/pubx-reports/american _educator/issues2005/cogsci.htm.

13 Oblinger, D. G. & Oblinger, J. L. (Eds) (2005). *Educating the Net Generation*. Boulder, CO: Educause. An Educause e-book retrieved 4/10/2006 from http://www.educause.edu/educatingthenetgen, pp. 2.4–2.7.

14 National Research Council. (1999). Bransford, J. D., Brown, A. L. & Cocking, R. R. (Eds.). *How People Learn. Brain, Mind, Experience, and School*. Washington, DC: National Research Council, National Academy Press, p. 55.

15 Johnson, D. W. & Johnson R. T. (1989). *Cooperation and Competition: Theory and Research*. Edina, MN: Interaction Book Co.

16 Webb, N. M. (1982). Student Interaction and Learning in Small Groups. *Review of Educational Research*, 52(3), pp. 421–445.

17 Springer, Stanne, & Donovan (1999).

18 Halpern, D. F. & Associates (1994). *Changing College Classrooms: New Teaching and Learning Strategies for an Increasingly Complex World*. San Francisco, CA: Jossey-Bass, pp. 152–153.

19 Hart-Landesberg, S., Braunger, J., & Reder, S. (1992). *Learning the Ropes: The Social Construction of Work-based Learning*. Berkeley, CA: National Center for Research in Vocational Education. ERIC document ED363726.

20 Czarnocha, B. & Prabhu, V. (2000). The Flow of Thought across the Zone of Proximal Development between Elementary Algebra and Intermediate English as a Second Language. *Proceedings of the 24th Conference of the International Group for the Psychology of Mathematics Education*, Hiroshima, Japan, Vol. 2; MacGregor, M. & Price, E. (1999). An Exploration of Aspects of Language Proficiency and Algebra Learning. *Journal for Research in Mathematics Education*, 30(4), pp. 449–467.

21 Meyers, C. & Jones, T. B. (1993). *Promoting Active Learning: Strategies for the College Classroom*. San Francisco: CA: Jossey-Bass, p. 24.

22 Simutis, L. (2001). The Future Isn't What It Used to Be. *ENC Focus*, 8(4), p. 17.

23 Burrill, G. et al. (2002). *Handheld Graphing Technology in Secondary Mathematics: Research Findings and Implications for Classroom Practice*. Report prepared through a grant to Michigan State University. Dallas, TX: Texas Instruments.

24 Mazur, E. (1997). *Peer Instruction: A User's Manual*. Upper Saddle River, NJ: Prentice Hall.

[25] U.S. Department of Education National Center for Education Statistics. (1999). Distance Education at Postsecondary Education Institutions: 1997–1998. (Rep. No. NCES 2000-013). Washington, DC: U.S. Department of Education.

U.S. Department of Education National Center for Education Statistics. (2003). The Condition of Education 2003. (Rep. No. NCES 2003-067). Washington, DC: U.S. Department of Education.

[26] Boettcher, J. V. (2003). Course Management Systems and Learning Principles Getting to Know Each Other. *Syllabus*, 16(12), pp. 33–36;

Meisner, G., & Hoffman, H. (2003). Leading the Way to Virtual Learning: The LAA physics laboratory. *Syllabus*, 16(11), pp. 26–28.

Novak, R. J. (2002). Using Benchmarking to Inform Practice in Higher Education. Benchmarking Distance Education. In B. E. Bender & J. H. Schuh (Eds.). *New Directions for Higher Education: No. 118*. San Francisco, CA: Jossey-Bass, pp. 79–92.

Reiser, R. A., & Dempsey, J. V. (2002). *Trends and Issues in Instructional Design and Technology*. Upper Saddle River, NJ: Pearson Education.

[27] Council of Regional Accrediting Commissions. (March 2001). Best Practices for Electronically Offered Degree and Certification Programs. Retrieved 4/12/2006 from http://www.wcet.info/resources/accreditation/Accrediting%20-%20Best%20Practices.pdf.

[28] Institute for Higher Education Policy. (2000). Quality on the Line: Benchmarks for Success in Internet-based Distance Education. Washington, DC: National Education Association. Retrieved 4/12/2006 from http://www.ihep.com/Pubs/PDF/Quality.pdf.

Beyond CROSSROADS

Chapter 8

Professionalism

Professionalism with its core values of expertise, autonomy, commitment, and responsibility[1] is at the heart of improving student success in mathematics. All two-year college mathematics faculty need to possess a strong academic preparation, participate in supportive professional development, exhibit the capacity for change and improvement, and shoulder the responsibility of carrying out multifaceted professional activities. Growing in knowledge of both mathematical content and pedagogical strategies, providing quality instruction, and exhibiting professional excellence are the results of deliberate faculty action.

> **Implementation Standard: Professionalism**
>
> Institutions will hire qualified mathematics faculty, and these faculty will engage in ongoing professional development and service.

Hiring and Mentoring Mathematics Faculty

One of the most important activities of a mathematics department is determining criteria for hiring new faculty. When choosing the best-qualified person for a vacancy, mathematics departments in two-year colleges should expect all candidates (full-time and adjunct) to have achieved the following:

- have earned at least a master's degree in mathematics or in a related field, which includes at least 18 hours of graduate-level mathematics[2]
- be knowledgeable in learning and teaching theory, the use of technology, and pedagogical strategies
- demonstrate evidence of professional involvement
- be familiar with and supportive of the mission of two-year colleges.

Candidates who have participated in teaching internships or in professional activities focusing on college teaching should be given particular consideration.[3] Understanding how students learn and applying appropriate and varied teaching strategies are important components of a mathematics faculty member's preparation.

> **Organizations providing higher learning must have qualified faculties—people who by formal education or tested experience know what students must learn—who create the curricular pathways through which students gain the competencies and skills they need.**
>
> North Central Association of Colleges and Schools, *The Higher Learning Commission Handbook of Accreditation,* 2005, p. 3-2-10.

In addition to general qualifications in mathematics, faculty who will teach specialized courses need specialized preparation in the following areas:

- developmental mathematics—knowledge of mathematics anxiety and associated coping strategies, motivation techniques, and student learning styles
- technical mathematics—hands-on experience in the workplace
- teacher preparation—solid grounding in pedagogy with connections to and experiences in the pre-K–12 school sector
- mathematics-intensive—an especially strong academic background in advanced mathematics
- statistics—an academic background in statistics that includes the study of at least two advanced statistical topics, such as multiple regression and analysis of variance.

Decreasing enrollments in undergraduate and graduate mathematics programs and competition from higher-paying technical careers present new and continuing challenges in recruiting and hiring qualified mathematics faculty. Because there is a critical shortage of qualified and capable applicants to fill the growing number of vacancies in mathematics departments, institutions and mathematics departments need to be proactive in the recruitment process to assure that the pool of applicants for both full-time and adjunct positions is sufficient and as diverse as possible. Diverse faculty contribute to social equity and also serve as vital role models for all students.[4]

Once hired, faculty new to the teaching profession should be provided with the orientation and mentoring needed to ease the transition into the institution and into the classroom. The orientation should also help them become familiar with departmental expectations and the needs of the college's diverse student population, and help them develop as leaders in their institution and in the professional mathematics community. "Mentoring is useful and powerful in understanding and advancing organizational culture, providing access to informal and formal networks of communication, and offering professional stimulation to both junior and senior faculty members."[5] Mentoring has a natural cycle of four parts and must be customized to meet individual needs:[6] (1) assignment and getting acquainted; (2) development of goals, procedures, and expected outcomes; (3) development of mutual confidence and satisfaction as goals are accomplished; and (4) discontinuation of mentoring when it is no longer needed. The mentoring process benefits both the mentor, who is often renewed, and the faculty member being mentored, who can become empowered. An effective mentor is a respected role model, a good listener, and a skilled communicator—flexible and responsive, informed and influential, encouraging and positive, and committed to the mentoring process.

Implementation recommendation: Two-year colleges will recruit, hire, orient, and mentor a qualified and diverse mathematics faculty.[7]

Actions to support this recommendation

Faculty actions:

- participate in developing criteria and hiring new faculty
- serve as mentors to other faculty.

Departmental/institutional actions:

- develop and apply suitable criteria for hiring new faculty
- be proactive in encouraging underrepresented groups to explore and engage in teaching careers
- advertise personnel vacancy notices widely; contact professional organizations, graduate schools, and other entities to broaden the applicant pool

- provide new faculty with activities and resources to ease their transition into the institution and the department
- offer a faculty mentoring program where qualified and experienced faculty mentor less experienced full-time and adjunct faculty
- provide opportunities for faculty to develop as leaders in their department, institution, and profession.

Professional Development and Service

Professional growth is the personal responsibility of each faculty member with support from the department, the college, and professional organizations. Professional development activities can be the key to fostering improvement in a mathematics department. Such activities enhance an instructor's mastery of content, knowledge of teaching, and self-esteem. By actively participating in faculty development, faculty can be aware of and implement major developments in content, pedagogy, and the effective use of technology. Effective teaching is a result of faculty preparation, experience, reflection, and continued professional development. Professional development may include sabbatical leave, as well as travel related to teaching, graduate coursework, group (or department) colloquia, individual study, learning about Internet resources, reading or writing scholarly journals, and virtual interest groups. These activities can result in an invigorated commitment to teaching and innovation, which benefits students, the department, the college, and society as a whole.

Mathematics departments and institutions should provide regular and comprehensive professional development programs serving both full-time and adjunct faculty. New full-time and adjunct faculty especially need to be encouraged to participate in departmental activities, discussions of curricular and pedagogical issues, and decisions regarding textbook selection. Regular department meetings involving full-time and adjunct faculty to discuss implementation of new or different instructional practices and program assessment promote change across the department. All too often, curricular and pedagogical change is driven by one person in the department. While one person *can* be a catalyst for change, the department may revert to an earlier status quo when the person leaves the institution. Lasting change requires the involvement of the entire department. Effective instructional change also requires involvement and commitment of the college administration supporting the department's goals and activities to maintain a quality mathematics education for all students.

Faculty can contribute to the profession by presenting workshops on topics such as teaching, learning, and curriculum design. They may also write articles for journals, reviews, textbooks, textbook supplements, or online materials. Faculty working together can contribute to the ongoing implementation of standards-based mathematics education. Engaging in discussions on curriculum issues, course development, teaching schedules, course delivery modes, and related topics help to establish curricular direction and priorities of the department. Equally important are discussions, both formal and informal, about teaching strategies to foster a supportive climate for student learning. Active involvement in campus-wide initiatives and service on college-wide committees build working relationships with colleagues from other departments. As faculty develop interdisciplinary courses and integrate student learning outcomes across disciplines, the academic community is strengthened, and students have more opportunities to appreciate the applications of mathematics.

> **Professional service fosters the building of a community of lifelong learners, who value expertise and encourage collaboration.**
>
> Ana Jiménez,
> *Project ACCCESS Fellow, Pima Community College, Tucson, Arizona,* 2005

Grantsmanship provides another avenue for mathematics faculty to engage in professional service. Successful grant applications can support research and other professional development activities, and fund the acquisition of needed instructional equipment. Two-year colleges can encourage faculty to work on grant proposals by awarding reassigned time for writing and providing grant personnel to assist with proposal preparation and grant administration. Once a grant has been received, the institution can support faculty with grant-funded reassigned time and summer remuneration.

Participation in professional organizations and societies is critical for professional growth. Professional organizations offer faculty numerous opportunities for professional development and promote enriching interaction and networking among mathematics educators across the country. Mathematics organizations also represent the interests of colleagues and the mathematics profession in national and international educational dialogues. At conferences offered by professional organizations, faculty can network with colleagues from other colleges, attend workshops and sessions helpful to their teaching practice, exchange ideas about teaching and learning, and examine the products and services exhibited by publishers and other companies serving the mathematics community. Funding professional development is always an ongoing personal and institutional challenge. Institutions can be proactive in providing internal funding, released time for faculty to pursue projects or attend professional meetings, and substitutes for missed classes.

Faculty have a stake in the betterment of the mathematics community at large. Building ties with faculty in K–12 schools is extremely useful, as is forming partnerships with business and industry. Team-teaching within colleges and between two-year and four-year institutions can strengthen the mathematical curriculum and teaching, and help foster an understanding of each institution's culture and environment.[8] Faculty can also help to build public support for mathematics education and work to diffuse the mathematics anxieties so prevalent in our society.

Implementation recommendations: Mathematics faculty will recognize that their professional responsibilities extend beyond the classroom. They will engage in professional activities within the department, the college, the mathematics education community, and outside communities to enhance mathematics curricula and instruction. Mathematics department chairpersons and college administrators will provide faculty with opportunities and support in their professional development efforts.

Actions to support these recommendations

Faculty actions:

- participate in significant professional development activities on a regular basis, including discussions about courses, programs, teaching methods, education research, and how to improve student learning in mathematics
- communicate the mathematical needs of students to faculty in other disciplines
- participate in consensus-building within the department and the college regarding student outcomes in mathematics courses and programs
- build collegiality and academic respect across disciplines for mathematics curricula and instruction
- be actively involved in professional organizations.

Departmental/institutional actions:

- hold regular meetings for full-time and adjunct faculty for sharing teaching strategies and review of curricula
- invite adjunct faculty to participate in departmental activities[9]
- provide faculty development activities to help faculty respond to major developments in content and pedagogy, including the effective use of technology[10]
- provide all full-time and adjunct faculty with reassigned time, substitutes, or financial support for professional development
- encourage the participation of underrepresented groups in professional activities.

The Scholarship of Teaching and Learning Mathematics

The "scholarship of teaching and learning (SoTL)" differs from traditional teaching in that it emphasizes teaching and learning as legitimate areas of scholarship. Faculty engaged in the scholarship of teaching and learning "frame and systematically investigate questions related to student learning—the conditions under which it occurs, what it looks like, how to deepen it and so forth—and do so with an eye not only to improving their own classroom but to advancing practice beyond it."[11] SoTL methods take many forms, ranging from simple critical observation of classroom patterns, to the use of classroom data to try out new classroom strategies, to research that compares testing methods to see which method best maximizes learning. The student is the focus of the teaching activity and the classroom becomes a laboratory in which data are collected and openly shared with peers for the purpose of improving the profession.

Documenting the achievement of mathematics students and the effectiveness of instructional strategies and using those results of that research to make valid improvements in mathematics programs can help to enhance student learning in mathematics. Self-assessment and professional reflection are powerful tools for improving teaching and learning. Periodic classroom assessments[12] can enable faculty to make critical modifications in instructional attitude, behavior, and content. The use of videotapes, audio recordings, portfolios, skills check lists, minute papers, and reflective journals can provide the faculty member with information to change positively and affect teaching performance. Collecting feedback from students regularly, assessing the effects of teaching, and making necessary adjustments are essential activities of the reflective practitioner.[13] To be credible and useful, the results must become public and be shared.

Teaching should be connected to the disciplinary and professional communities in which faculty pursue scholarly work.[14] A key feature of the scholarship of teaching is the commitment of faculty to make their practices public, documenting their pedagogical work, and putting it forward for review. Teaching should be viewed as a process of ongoing reflection and inquiry that requires collegial exchange and openness.[15] Mathematics faculty should be proactive in comparing results of the assessment of their students' learning outcomes with those of similar classes in the department. Strategies to communicate the findings from the scholarship of teaching include the use of the following resources:

When we independently explore professional development opportunities, we nurture our own desire to learn and our excitement in the learning process…

Thoughtfully setting personal and professional priorities and then regularly reflecting upon these priorities is a vital aspect of continuing professional enrichment.

AMATYC, *Opportunities for Excellence: Professionalism and the Two-Year College Mathematics Faculty,* 2001, p. 48.

- informal discussions among peers promoting excellence in teaching
- journals focusing on mathematics pedagogy that serve as vehicles for instructors to share what they know and do[16]
- newsletters for fellow practitioners (departmental, regional, state, or national)
- a collection of essays by faculty who are investigating teaching and learning in mathematics to document investigative work by sharing methods, approaches, reflection, and analysis and to provide a model for others
- Web sites that serve as clearinghouses for effective pedagogical practices
- digital and multimedia devices to demonstrate instructional strategies.

Faculty who persistently ask "Why?" even in ordinary circumstances develop a deeper, richer understanding as their research evolves. Applying the Implementation Cycle of *Beyond Crossroads* to a project such as implementing an innovative teaching method is a way for faculty to engage in the scholarship of teaching.

Implementation recommendations: Mathematics faculty will regularly engage in empirical research on their teaching and share the results with peers for the purpose of improving student learning in mathematics. Faculty will use reflective self-assessment of their teaching to develop and refine teaching strategies and to assess the impact of those techniques on student learning.

Actions to support these recommendations

Faculty actions:

- stay abreast of research in mathematics and mathematics education
- engage in educational research to document the effectiveness of their instructional strategies
- communicate the results of this research broadly.

Departmental/institutional actions:

- encourage and support innovation in mathematics classrooms
- provide incentives for specialized training and to encourage innovation, classroom research, and the scholarship of teaching projects
- support faculty, both full-time and adjunct, in research projects and professional development activities through reassigned time or stipends.

Improving Student Learning through Faculty Evaluation

Faculty evaluation is the process of self-review, as well as the review of faculty work by supervisors, peers, and students. Faculty evaluation provides insight to improve instruction and grow professionally, in addition to being a basis for personnel decisions. A clear distinction should be made between assessment of student learning for the purposes of course or program improvement and the evaluation of faculty.

Different types of evaluation, such as peer evaluation, student evaluation, self-evaluation, and administrative evaluation, may contribute to a faculty evaluation process. Each type of evaluation is a valid tool for self-improvement in teaching and learning. Objective and subjective criteria should be included in the evaluation process. Informal discussion among and between peers should be encouraged to promote excellence in teaching. Students should have the opportunity to give feedback to faculty at multiple times throughout a course, with the expectation that faculty will use this information to improve student learning. An effective faculty evaluation system should include the following components:

- significant faculty input is sought as the evaluation process is developed
- content and form of the evaluation process are mutually agreed upon by faculty and administration
- multiple tools for evaluation are available and used
- written documentation outlining the aspects and requirements of the evaluation are given to faculty prior to implementation of the evaluation system
- evaluations are conducted in similar ways for all faculty in the institution
- a description of the evaluation process to be used for faculty personnel decisions, such as promotion or tenure, is clearly communicated.

The periodic evaluation process for adjunct faculty should be as rigorous as that for full-time faculty. The tools for evaluation may have to be slightly modified for use with adjunct faculty based on the contractual agreement made with the adjunct faculty regarding teaching, conducting office hours, and assisting students.

> **Evaluation can be a positive force when used to encourage community college faculty members to continue their professional growth and thereby improve the delivery of their professional services.**
>
> University of Hawaii Community Colleges, *Procedures for Evaluation of Faculty*, April 1990

Implementation recommendation: Faculty evaluation will be regular, systemic, and ongoing, based on criteria known to all faculty, with the goal of improving student learning in mathematics.

Actions to support this recommendation

Faculty actions:

- engage in self-review and reflection to grow professionally and improve student learning
- develop and assess a personal professional development plan
- work together with administrators to develop the components and criteria of evaluation for full-time and adjunct faculty
- use the results of faculty evaluation to improve student learning in mathematics.

Departmental/institutional actions

- provide faculty with a description of the faculty evaluation process
- provide input from multiple sources in the faculty evaluation process.

Conclusion

Faculty grow and improve professionally, implement change, and embark on informed instructional decision-making when all essential elements of a system are engaged and operating in concert. Faculty need to accept and take responsibility for continuing to learn about mathematics and effective mathematics instruction. Departments and institutions can foster faculty growth and change by providing financial support and the time faculty need to develop and implement effective strategies. Planning and implementing improvements can be facilitated by using the *Beyond Crossroads* Implementation Cycle. Administrators, rather than maintaining the status quo, can encourage reform and support efforts to enhance student learning.[17] Leadership and material resource management, policy, curriculum, instruction, and assessment must be aligned, with the goal of optimizing student and instructional outcomes.[18]

HIGHLIGHTS Implementing the Standard for Professionalism

Professionalism

Institutions will hire qualified mathematics faculty, and these faculty will engage in ongoing professional development and service.

At a standards-based institution, the *faculty*

- ☑ view mathematical and pedagogical knowledge as dynamic, requiring lifelong learning.
- ☑ use research to make informed decisions about instructional practices.
- ☑ actively participate in professional development activities and service.
- ☑ document results of their classroom activities and innovations, and communicate the results broadly.
- ☑ engage in ongoing self-reflection of one's teaching.

At a standards-based institution, the *mathematics department* and *the institution*

- ☑ assure that mathematics faculty participate in the hiring process to find capable and qualified faculty.
- ☑ hold regularly scheduled department meetings to discuss the teaching and learning of mathematics.
- ☑ invite adjunct faculty to participate in department meetings and discussions.
- ☑ encourage and support faculty professional development.
- ☑ provide professional development activities for all faculty as a part of department meetings or at special faculty meetings.
- ☑ provide mentors to faculty who need them and provide orientation for new faculty.
- ☑ solicit faculty input in the design of effective faculty evaluation processes.

1 Strauss, G. (1963). Professionalism and Occupational Associations. *Industrial Relations: A Journal of Economy and Society*, 2(3), pp. 7–9.

2 AMATYC. (1992). *Guidelines for Academic Preparation of Mathematics Faculty Two-Year Colleges*.

3 Examples of current programs: Project ACCCESS (Advancing Community College Careers: Education, Scholarship, and Service), jointly sponsored by AMATYC and the MAA and funded by the ExxonMobil Foundation, is a mentoring and professional development initiative for two-year college faculty http://www.maa.org/Project ACCCESS. The PMET (Preparing Mathematicians for Educating Teachers) initiative was designed to help mathematicians enhance the teaching of mathematics courses for future teachers, sponsored by the MAA. PREP (Professional Enhancement Program) is a comprehensive, professional career enhancement project of the MAA funded by NSF that offers a large number of workshops and short courses across the country http://www.ma.org/prep. Project NExT (New Experiences in Teaching), sponsored by the MAA and funded by the ExxonMobil Foundation, is a professional development program for new or recent Ph.D.s in the mathematical sciences (including pure and applied mathematics, statistics, operations research, and mathematics education) http://archives.math.utk.edu/projnext. The College Faculty Preparation Program (CFPP) is a discipline-specific program to better prepare graduate students interested in teaching careers at the community college or university level. Humboldt State University offers a graduate certificate in college teaching in the area of mathematics http://humboldt.edu/~gradst/cfpp/Programs/ProgramsMathematics.html.

4 To reach underrepresented minorities, such publications as the *Quarterly Newsletter* of the National Association of Mathematicians may be helpful http://www.math.buffalo.edu/mad/NAM/index.html.

5 Luna, G. & Cullen, D. L. (1995). *Empowering the Faculty: Mentoring Redirected and Renewed. ERIC Digest*, ERIC document ED399888.

6 Cook, S. G. Practical Tips for Effective Faculty Mentoring. Reprinted in the University of Wisconsin–Milwaukee Employee Newsletter, Vol. 1, No. 2, April 2004. Originally printed in *Women in Education*, December 2003.

7 AMATYC. (1989). Reaffirmed 2005. *Position Statement on Equal Opportunity in Mathematics*.

8 AMATYC. (1996). *Guidelines for Internships for Two-Year College Mathematics Faculty*.

9 AMATYC. (1998). *Position Statement on Support for Professional Development*.

10 American Mathematical Association of Two-Year Colleges (AMATYC). Cohen, D. (Ed.). (1995). *Crossroads in Mathematics: Standards for Introductory College Mathematics before Calculus*. Memphis, TN: American Mathematical Association of Two-Year Colleges.

National Council of Teachers of Mathematics (NCTM). (2000). *Principles and Standards for School Mathematics*. Reston, VA: National Council of Teachers of Mathematics.

11 Hutchings, P. & Shulman, L. S. (1999). The Scholarship of Teaching: New Elaborations, New Developments. *Change*, 31(5), p. 12.

12 In these classroom assessment techniques, students may be asked to summarize an important concept discussed in class in a one-minute paper or describe a concept that is still “muddy” or unclear at the end of class.

13 Angelo, T. & Cross K. P. (1993). *Classroom Assessment Techniques, A Handbook for College Teachers*, Second Edition. San Francisco, CA: Jossey-Bass.

Angelo, T. Presentation at the AMATYC Annual Conference, Salt Lake City, UT, November, 2003.

14 Shulman, L. S. (1993). Teaching a Community Property: Putting an End to Pedagogical Solitude. *Change*, 25(6), pp. 6–7.

15 Hutchings, P. (Ed.) (2000). *Opening Lines: Approaches to the Scholarship of Teaching and Learning*. Stanford, CA: The Carnegie Foundation for the Advancement of Teaching.

Hutchings, P. (Ed.) (1998). *The Course Portfolio: How Faculty Can Examine Their Teaching to Advance Practice and Improve Student Learning*. Washington, DC: American Association for Higher Education.

16 Examples: *The Journal of Scholarship of Teaching and Learning* (JoSoTL), a Web journal, serves as an electronic forum for sharing scholarly work, retrieved 4/12/2006 from www.iusb.edu/~josotl/. In 1998, The Carnegie Foundation for the Advancement of Teaching initiated CASTL: the Carnegie Academy for the Scholarship of Teaching and Learning. CASTL aims to foster significant, long-lasting learning for all students and to enhance the practice and profession of teaching: www.carnegiefoundation.org/CASTL.

17 Sparks, D. (2002). Dreaming All That We Might Realize. *ENC Focus*, 9(1).

18 Long, M. J. (1997). Things to Consider When Implementing Reforms in the Mathematics Classroom. *Reform in Math and Science Education: Issues for Teachers*. Columbus, OH: Eisenhower National Clearinghouse.

Chapter 9

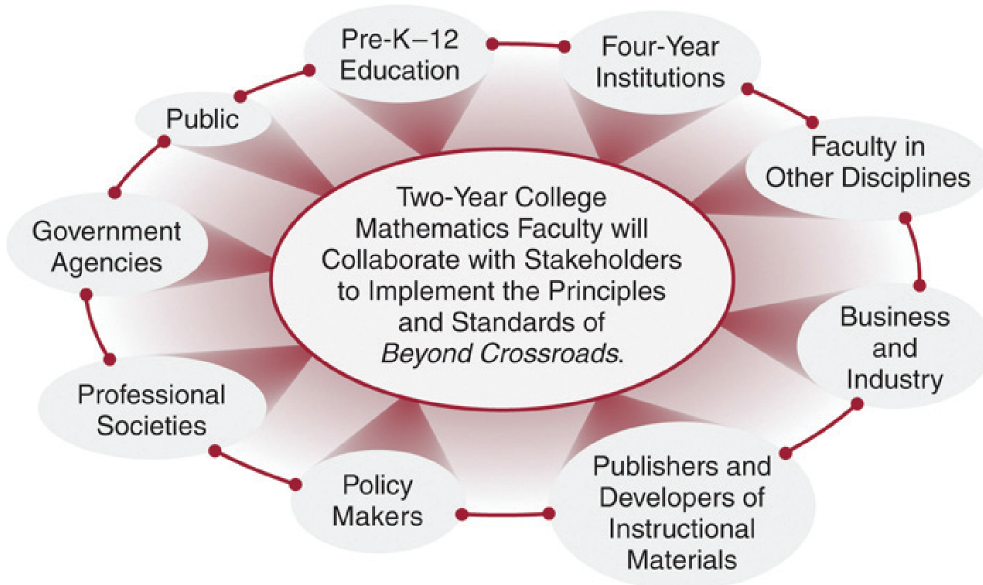
Involvement of Stakeholders in Implementation

The complex issues of learning, assessment, curriculum, teaching, and professionalism can only be fully addressed through actions of all stakeholders, not just a single individual or a single entity. Faculty at two-year colleges need to work with their institution and other faculty, departments, institutions, and stakeholders to improve mathematics programs in the first two years of college to influence and enhance mathematics education for all students and to respond to the mathematical needs of the community. The principles and standards outlined in this document are best served when two-year college mathematics faculty and institutions collaborate with the following entities:

- the mathematics community to build public understanding of and support for improvements in mathematics education
- pre-K–12 institutions and four-year institutions to align exit and entrance requirements, instructional strategies, and curricula
- faculty in other disciplines to infuse mathematics across the curriculum
- publishers and developers of instructional materials to create standards-based instructional resources
- business and industry so that desired employee skills and strategies for achieving them are outlined and incorporated into mathematics courses and programs
- professional societies, government agencies, and educational institutions to build consensus and provide guidance to practitioners

The vision of *Beyond Crossroads* includes an action plan. The activities of the plan are led by mathematics faculty at two-year colleges collaborating with all stakeholders to improve student learning in mathematics in the first two years of college. Implementing the standards requires the involvement of a mathematics community, functioning purposefully as a whole to improve student learning in mathematics. "Systemic change requires new forms of partnerships to make the system more productive and to provide solutions that cut across the system components."[1] Inherent in such collaborative relationships is a willingness to work together to build consensus and a commitment to put standards into practice with systemic actions. The principles and standards of *Beyond Crossroads* are put into action by two-year college mathematics faculty collaborating with the stakeholders shown in the following figure.

Figure 4 Collaborating with Stakeholders

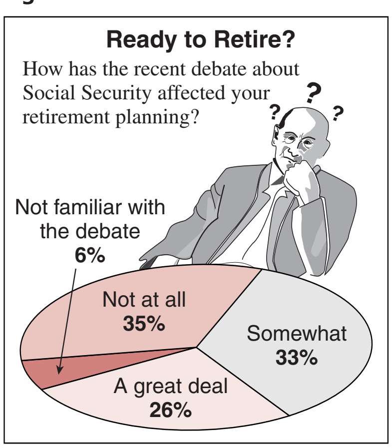


Building Public Understanding and Support

The general public can play an important role in improving mathematics education in the United States. However, in order to help the public understand the many challenges facing mathematics faculty, the mathematics community needs to actively communicate the critical role of quantitative literacy for all citizens and the reasons for changes in mathematics classrooms. "Math literacy—and algebra in particular—is the key to the future of disenfranchised communities... ."[2] The ability to reason analytically and do mathematics impacts on the following:

- an individual's career options in a global and technological society
- a consumer's ability to make decisions and solve problems (e.g., comparison shopping for a car loan or mortgage loan
- a citizen's ability to choose between various policy options (e.g., comparing environmental options and where to use limited funds).

Figure 5



Many technical careers require significant application of mathematics. Employees in business, industry, and government are expected to apply mathematics skills and concepts when making informed decisions and then use the language of mathematics to communicate those decisions. Describing and interpreting data requires mathematical analysis, representations, and manipulations, although the users seldom identify themselves as "doing" mathematics. As many businesses and industries experience a shortage of technically trained workers, these workers are being recruited from overseas or the work is completely outsourced overseas. To be economically competitive and secure, the United States needs citizens who are mathematically literate and capable of filling these jobs.

Acquiring quantitative skills and mathematics power is essential, not optional. Quantitative thinking is a daily activity. Teaching professionals need to recognize that local and national newspapers, news magazines, and professional journals expect their readers to read and interpret tables, graphs, and charts, as Figure 5 shows. An informed citizen should understand mathematical statements such as "The margin of error is plus or minus 5%." Analyzing data and evaluating the validity of claims require quantitative thinking and mathematical reasoning. The public needs to understand the nature and scope of mathematics today and the importance of quantitative literacy for all.

In addition, research about how students learn mathematics and the rewards of active student learning needs to be shared beyond the classroom. Many students and parents are suspicious of innovative or new teaching methods and the use of technology. The mathematics community must recognize these suspicions and communicate the results of research that show the benefits of active student learning, the use of technology, and alternative delivery formats.

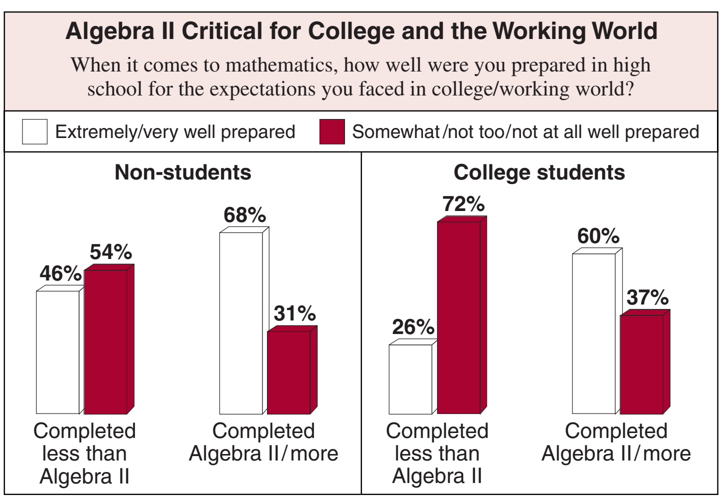
Implementation recommendations to build public understanding and support:

- The mathematics community will communicate the nature and scope of mathematics and the importance of quantitative literacy to the public and share the results of research on the learning and teaching of mathematics with the public.
- Mathematics faculty and mathematics organizations will listen to, analyze, understand, and address the public's questions and concerns about mathematics education and quantitative literacy and will provide opportunities for parents, faculty from other disciplines, administrators, and public officials to gain a deeper understanding of the benefits of standards-based mathematics instruction.
- The mathematics community will seek opportunities to provide input on mathematics used in public interest media.

Connecting with pre-K–12 Education

Dialogue and collaboration between pre-K–12 school districts and institutions of higher education are essential to develop and maintain a consistent, positive, and significant mathematical experience for all students. Institutions that collaborate benefit from each others' resources. Two-year college faculty can learn from middle and high school faculty who use integrated curricula. Two-year college teacher preparation programs can benefit by incorporating pre-K–12 classroom experiences into their courses.

Figure 6



How Prepared Are Public High School Graduates?/Hart Research & Public Opinion Strategies for Achieve, Inc.

"The academic intensity of the student's high school curriculum still counts more than anything else in precollegiate history in providing momentum toward completing a bachelor's degree."[3] And while taking more mathematics in high school is critical, "It's not merely getting beyond Algebra II in high school any more. The world demands advanced quantitative literacy, and no matter what a student's postsecondary field of study—from occupationally oriented programs through traditional liberal arts—more than a ceremonial visit to college-level mathematics is called for."[4]

Collaborative efforts are necessary to achieve continuity in pre-K–12 and higher education mathematics curricula and instructional strategies. In particular, pre-K–12 and higher education mathematics faculty need to collaborate on the content of mathematics placement tests. Student mathematics outcomes should be aligned across institutions so tests are fair, reasonable, and reflect agreed upon outcomes. Many students in high school experience mathematics in context, using technology. A majority of high school exit examinations allow the use of graphing calculators.[5] In contrast, some higher education mathematics placement exams test only basic arithmetic and algebraic computation without technology. These differences in the use of technology also need to be addressed. Collaborative efforts to implement standards-based mathematics can be an initial step in minimizing the need for remediation in postsecondary mathematics education, address the critical need for students to complete algebra, and help to ease the transition from high school to college.[6]

Providing in-depth professional development for pre-K–12 mathematics teachers to implement mathematics standards for school mathematics has become a priority in many schools. The need is particularly great in the middle grades, where a high percentage of teachers have mathematics preparation that focused mainly on computational arithmetic. These same teachers, with minimal training, are expected to lay the foundations of algebra, teach basic geometry and measurement concepts, and introduce applications of probability and elementary statistics to students in the middle grades. Two-year colleges and school districts should partner, working together to help teachers better understand the mathematics content and related pedagogy that local and state standards require.

Dual enrollment programs—programs where a high school student enrolls in a postsecondary institution and may earn credit in both institutions simultaneously—promote connections between high schools and postsecondary institutions. Maintaining appropriate content and pedagogy in these programs is challenging. The high school mathematics courses may be taught on a college campus or collegiate courses may be taught in high schools. Regardless of the dual enrollment model used, mathematics faculty and institutions should agree on student outcomes in mathematics courses and programs, and develop appropriate curricula and instructional strategies that enhance the mathematical knowledge of all students.

Implementation recommendations to connect pre-K–12 schools with higher education:

- Two-year college and pre-K–12 district personnel will engage in dialogue and collaborate to develop a continuous, seamless, and coherent mathematical experience for students.
- College, university, and high school mathematics faculty will work together to assure that students are able to make the transition from high school to postsecondary education as easily as possible, aligning high school exit competencies in mathematics with higher education entrance requirements and content on mathematics placement tests.
- Two-year college and pre-K–12 district personnel will work together to address the need for the continuing education of pre-K–12 teachers in mathematics.

Connecting with Four-Year Institutions

Open communication between two-year and four-year institutions is a prerequisite for successful transition for the large numbers of students transferring between the institutions. As a result, it is critical these institutions collaborate to build mathematics programs and develop articulation agreements that address issues including entrance and exit requirements, course content, pedagogy, the use of technology, and assessment strategies.

Articulation agreements are essential for program integrity and to decrease duplication in course content for students. Effective articulation agreements are the result of ongoing communication and consensus-building activities. Assessment strategies and instruments, exit and entrance requirements, instructional strategies such as collaborative learning, writing assignments, the use of technology, and the inclusion of authentic applications, must be agreed upon and aligned. Expectations of students and faculty must be communicated broadly and implemented at all institutions.

Two-year colleges play a vital role in the preparation of future teachers. It is critical for two-year colleges to "develop partnerships with both K–12 schools and four-year colleges and universities to design curriculum and assessments, enhance K–12 and community college student progress in science, math and technology, and maximize scarce public resources through creative use of shared technologies, classroom space, and staff expertise."[7] Two-year college mathematics faculty, for example, can learn lessons from teacher preparation programs and provide hands-on experiences in schools for future teachers.

Successful transfer from a community college to a four-year institution is often the only opportunity these individuals (community college transfer students) have to achieve a bachelor's degree, particularly in the case of low-income students. If articulation programs are not in place, these high-risk students often fall through the cracks and never complete their education.

Education Commission of the States, *Transfer and Articulation Policies,* February 2001, p. 1.

Implementation recommendation to connect two-year colleges with four-year institutions:

- Mathematics faculty at two-year colleges and the four-year institutions will collaborate to build and enhance mathematics programs and develop articulation agreements regarding exit and entrance requirements, course content, pedagogy, the use of technology, and assessment strategies to ensure that students will experience smooth transitions from one institution to another.

Connecting with Publishers and Instructional Resource Developers

The mathematical and instructional standards and strategies presented in *Beyond Crossroads* need to be communicated broadly and incorporated into instructional materials that may be in print, video, digital, or some other media or format. Incorporating the recommendations of *Beyond Crossroads* into instructional materials benefits all parties—students, faculty, and publishers. Faculty, collaborating with publishers, as both writers and users of standards-based materials, can incorporate research into practice and guide the design of textbooks and other instructional materials. Only through cooperation between faculty and publishers, will standards-based materials become the norm.

To get texts to change, faculty must tell publishers' representatives what they want and what they need.

AMATYC, *Mathematics for the Emerging Technologies,* 2003, p. 15

Implementation recommendations to connect faculty with publishers and instructional resource developers:

- Mathematics faculty, AMATYC, and its members will collaborate with publishers and other course materials developers to incorporate the principles and standards of *Beyond Crossroads* into instructional materials.
- AMATYC and its members will collaborate to create digital products to give faculty access to ideas, programs, and materials that support *Beyond Crossroads*.

> **Together, we must ensure that U.S. students and workers have the grounding in math and science that they need to succeed and that mathematicians, scientists and engineers do not become an endangered species.**
>
> Business Roundtable, *Tapping America's Potential: The Education for Innovation Initiative*, July 2005, p. 14.

Connecting with Business and Industry

Two-year colleges, business, and industry need to collaborate and identify the mathematical needs of the workplace in the 21st century. Employees are now expected to be quantitatively literate and possess a high level of specific mathematical skills. Higher education institutions, especially two-year colleges, play an important role in educating and training those employees.

Mathematics departments play a major role in the mathematics preparation of the workforce. In order to be responsive to workforce needs, departments should establish advisory committees including representatives from business and industry to engage in regular conversations about the mathematical expectations of prospective employers. These committees should share useful information regarding content and assessment, identify opportunities for recruiting guest speakers and adjunct faculty with unique qualifications, and ways to understand the language and culture of education and business.

Business and industry partnerships must provide opportunities for offering specialized mathematics courses and programs for the workforce and aligning these courses and programs with employee expectations. Institutions need to find ways to implement methods for tracking their students after enrollment or graduation to determine whether or not mathematics courses and programs meet students' educational or career needs.[8]

Implementation recommendations to connect education with business and industry:

- Mathematics faculty will establish relationships with business and industry to gather information about workforce needs and to incorporate relevant content and application into their courses.
- Two-year colleges will work with other institutions of higher education to collect information to determine whether or not mathematics courses and programs have met students' career needs and whether they have acquired the necessary mathematics skills needed by their employers.

Connecting with Professional Societies, Government Agencies, and Policy Makers

Designing and implementing policy in mathematics education is the responsibility of all stakeholders. These policies should be determined after careful dialogue and active involvement among them. The mission and uniqueness of each entity should be respected, while putting forth a unified voice in support of standards-based mathematics education and improvement of student learning in mathematics.

All stakeholders need to continue to establish and enhance their relationships. National leaders in government, education, and on professional boards need to be informed of *Beyond Crossroads*, incorporate its principles and standards into policy, and allocate funds to promote innovation. AMATYC and its members should also stay informed about activities and documents of other stakeholders. The notable contributions in mathematics education of the National Council of Teachers of Mathematics and the Mathematical Association of America influenced this document. Garnering support from professional organizations, federal agencies, policy makers, foundations, and businesses will significantly help to further implement *Beyond Crossroads*.

> **America's economic preeminence, national security, and social stability are dependent on the mathematics and science abilities of its citizens.**
>
> Business–Higher Education Forum, *A Commitment to America's Future: Responding to the Crisis in Mathematics and Science Education*, January 2005, p. 32

Implementation recommendations to connect professional societies, government agencies, and policy makers with education:

- AMATYC and its members will collaborate with other boards, societies, organizations, agencies, and policy makers to disseminate *Beyond Crossroads* broadly and promote standards-based mathematics education for all students in the first two years of college.
- AMATYC and its members will actively work with government agencies, foundations, and other funding organizations to align AMATYC's programs with the mathematics community and they will partner with other professional mathematics organizations to create a unified strategy for improving mathematics education for all students:

Conclusion

Improving student learning in mathematics and implementing mathematics standards is a multifaceted endeavor. Globalization, technological advances, and dynamic mathematical expectations and requirements, create ongoing challenges for the mathematics education community. Faculty, with support from their institutions, shoulder the day-to-day responsibility for continuing to grow in their mathematical and pedagogical knowledge, contributing to their profession, addressing the learning needs of their students, and preparing quantitatively literate citizens for the future. But, faculty and institutions cannot accomplish their goals alone. The collection of individuals and institutions dedicated to improving mathematics instruction will become a community working towards systemic change when every stakeholder collaborates to achieve the following:

Change is good…you go first.
Bumper sticker

- build public understanding and support
- make connections with pre-K–12 school districts and higher education institutions
- develop standards-based instructional resources
- establish an ongoing dialogue with business and industry
- speak with a unified voice in support of the improvement of mathematics education for all.

Implementing standards is not easy. Improving student learning in mathematics cannot occur only within one isolated entity. It requires incremental and systemic changes in actions, structures, and mechanisms. These changes lead to the most effective outcome when individuals and institutions design and implement cultures, strategies, and policies collaboratively to improve student learning in mathematics. This implementation process continues to improve through reflection, assessment, redefinition of goals and objectives, and continuous improvement. Embracing change requires working together, making informed decisions, using innovative strategies, documenting results, and refining goals, objectives, and actions. If all stakeholders do not play an active role in implementation, the impact of those who do, can be greatly diminished. Everyone has a role in creating a mathematics community with its core goal of helping all students be successful learners in mathematics.

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1 Leinwand, S. & Burrill, G. (Eds). (2001) *Improving Mathematics Education: Resources for Decision Making*, Washington, DC: National Academy Press, p. 45.

2 Moses, R. (2001). *Radical Equations: Math Literacy and Civil Right*s. Boston, MA: Beacon Press, p. 5.

3 National School Boards Foundation. (2005). Education Leadership Toolkit: Change and Technology in America's Schools. A project of the National School Boards Foundation implemented by the NSBA Institute for the Transfer of Technology to Education with a grant from the National Science Foundation (REC-9603345), retrieved 3/30/2006 http://www.nsba.org/sbot/toolkit/chned.html, p. xviii.

4 Adelman, C. (2006). *The Toolbox Revisited: Paths to Degree Completion from High School through College*. Washington, DC: U.S. Department of Education, p. 108. Retrieved 4/12/2006 from http://www.ed.gov/rschstat/research/pubs/toolboxrevisit/index.html.

5 Eighty-one percent of the states in the U.S. require exit exams that allow calculators to be used on at least some test items. Similarly, 79% of the states without exit exams allow calculators to be used on state-mandated mathematics tests for high school students. *NCTM News Bulletin* (Sept. 2003), p. 8.

6 Achieve, Inc. and the National Governors Association. (2005). *An Action Agenda for Improving America's High Schools: 2005 National Education Summit on High Schools*. Washington, DC: Achieve, Inc.

7 American Association of Community Colleges (AACC). (2005). *Teaching by Choice: Community College Science and Mathematics Preparation of K–12 Teachers*. Washington, DC: Community College Press, p. 15.

8 Discussions with Dr. Sadie Bragg, Borough of Manhattan Community College (NY) and Dr. Judy Ackerman, Montgomery College (MD) (August 2003).

Chapter 10

Moving from Vision to Reality

The Implementation Standards of *Beyond Crossroads* move toward reality when all stakeholders adopt the principles and standards as their own, challenge themselves to translate that knowledge into practice, and make improvements on a continuing basis. Mathematics faculty must lead the way in this implementation process. However, they cannot do it alone. Each stakeholder needs to make a commitment to dialogue and collaborate with others, be informed about research on learning and teaching mathematics, and continually engage in activities to improve the learning and teaching of mathematics.

> **It is easy to be complacent about U.S. competitiveness and preeminence in science and technology ...But the world is changing rapidly, and our advantages are no longer unique... we are obliged to renew those commitments in education, research, and innovation policies...**
>
> The National Academies, *Rising Above the Gathering Storm,* 2005, ES-8.

Beyond Crossroads presents a renewed vision for mathematics courses and instruction in the first two years of college. It does not outline detailed curricula for each department or prescribe instructional methods for every professional. Instead, the focus is on making informed decisions and creating an appropriate student-centered learning environment for all students based on what is known about how students best learn mathematics. The vision is that each student achieves ambitious learning outcomes in mathematics and every faculty member pursues professional growth as a life-long continual process. Embracing change and moving outside one's comfort zone may not be always be easy, but are essential components of being a professional. Building upon and extending the standards set forth in the 1995 *Crossroads*, this document puts research into practice with five implementation standards to improve mathematics education for all students:

Implementation Standard: Student Learning and the Learning Environment

Mathematics faculty and their institutions will create an environment that optimizes the learning of mathematics for all students.

Understanding *how* students learn and creating a learning environment that maximizes student learning in mathematics requires the active involvement of every faculty member and each component of the institution.

Implementation Standard: Assessment of Student Learning

Mathematics faculty will use the results from the ongoing assessment of student learning of mathematics to improve curricula, materials, and teaching methods.

Assessment of student learning at the class, course, and program levels of instruction should be linked to student learning outcomes. Assessment tools should be designed to measure what is important for students to learn.

Implementation Standard: Curriculum and Program Development

Mathematics departments will develop, implement, evaluate, assess, and revise courses, course sequences, and programs to help students attain a higher level of quantitative literacy and achieve their academic and career goals.

The direction of curriculum and program development should be towards designing courses and programs that build mathematical understanding by actively engaging students and teaching quantitative concepts and skills that achieve each student's academic and career goals.

Implementation Standard: Instruction

Mathematics faculty will use a variety of instructional strategies that reflect the results of research to enhance student learning.

Effective mathematics instruction requires a variety of resources, materials, technology, and delivery systems that consider students' diverse learning styles and instructors' diverse teaching styles.

Implementation Standard: Professionalism

Institutions will hire qualified mathematics faculty, and these faculty will engage in ongoing professional development and service.

Professionalism with its core values of expertise, autonomy, commitment, and responsibility is at the heart of improving students' learning in mathematics.

The standards-based mathematics outlined in the previous chapters envisions each education professional as knowledgeable about the learning and teaching of mathematics, using that knowledge to make informed decisions within the context of the individual needs of students, the capabilities of each faculty member, and the mission of the institution. This document advocates an approach to teaching mathematics in the first two years of college that includes making incremental or comprehensive systemic changes—whichever are necessary. These actions require a collaborative effort on the part of all stakeholders. The Implementation Cycle of *Beyond Crossroads* in Chapter 3 outlines a process for implementing these changes. When goals and objectives are defined based on input from stakeholders, and activities implemented, evaluated, and refined based on results, continuous improvement results.

> **The important thing is not to stop questioning.**
> Albert Einstein

Teaching and learning mathematics is a dynamic process. Each student is unique, as is each faculty member. Each department and institution has its own culture. Embracing change can be challenging, but making the transition from teacher to professional involves taking the first step and then continuing the improvement process. Professionals plan, implement, and reflect regularly on their practice to improve student learning. Engaging in constant professional growth can make teaching more interesting and enjoyable.

The Implementation Standards of *Beyond Crossroads* renew and extend the goals, principles and standards set forth in 1995 in *Crossroads*. All roads lead to the improvement of student learning in mathematics and the professionalism of mathematics faculty. In moving from vision to reality, each faculty member is an informed professional who embraces change, explores, experiments, and makes improvements in his/her classroom as a natural state and every student has improved his/her quantitative literacy and workplace skills and maximized his/her success in mathematics in the first two years of college.

> *"We believe this standards-based reform effort will provide all students with a more engaging and valuable learning experience. Our students deserve no less; our nation requires no less; and we must demand no less of ourselves."*[1]

We also believe that ongoing professional growth is the key to a rewarding, enjoyable career in teaching. So let us all grow, learn, enjoy...teach!

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1 American Mathematical Association of Two-Year Colleges (AMATYC). Cohen, D. (Ed.). (1995). *Crossroads in Mathematics: Standards for Introductory College Mathematics before Calculus*. Memphis, TN: American Mathematical Association of Two-Year Colleges, p. 69.

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Sidebars and Figures

Page 1: American Mathematical Association of Two-Year Colleges (AMATYC). (1995). Cohen, D. (Ed.). *Crossroads in Mathematics: Standards for Introductory College Mathematics Before Calculus*. Memphis, TN: American Mathematical Association of Two-Year Colleges, Chapter 1, cover page.

Page 3: Caballero, J. (Fall 1989). Everybody a Mathematician? *CAIP Quarterly*, 2(2), p. 2.

Page 9: National School Boards Foundation. (2005). *Education Leadership TOOLKIT: Change and Technology in America's Schools, Change and Education*. A project of the National School Boards Association's Institute for the Transfer of Technology to Education with a grant from the National Science Foundation (REC-9603345), retrieved 3/30/2006 http://www.nsba.org/sbot/toolkit/chned.html.

Page 17: Oesterreich, H. (2000). *Characteristics of Effective Urban College Preparation Programs*, ERIC Digest Number 159, p. 4.

Page 19: Shaw, P. S. (1997). *An Analysis and Evaluation of Policies and Practices of Student Placement into College Algebra Classes at Paradise Valley Community College*. Unpublished doctoral dissertation, Nova Southeastern University, Fort Lauderdale-Davie, FL. p. 6.

Page 20: American Mathematical Association of Two-Year Colleges (AMATYC). (2003). Hovis, M. A., Kimball, R., & Peterson, J. (Eds.). *A Vision: Mathematics for the Emerging Technologies, A Report from the Project Technical Mathematics for Tomorrow: Recommendations and Exemplary Programs. Memphis*, TN: American Mathematical Association of Two-Year Colleges, p. 9.

Page 22: Polya, G. (1973). *How to Solve It—A New Aspect of Mathematical Method*, Second Edition. Princeton, NJ: Princeton University Press, p. v.

Page 23: Tobias, S. (June 1, 1991). Math Mental Health. *College Teaching*, 39(3), p. 93.

Page 25: National Council of Teachers of Mathematics (NCTM). (2000). *Principles and Standards for School Mathematics*. Reston, VA: National Council of Teachers of Mathematics, p. 18.

Page 30: U.S. Department of Education. (2000). *Before It's Too Late: A Report to the Nation from the National Commission on Mathematics and Science Teaching for the 21st Century*. Washington, DC: US Department of Education, p. 22.

Page 32: Angelo, T. & Cross, P. (1993). *Classroom Assessment Techniques, A Handbook for College Teachers*. Second Edition. San Francisco, CA: Jossey-Bass, p. 4.

Page 33: National Council of Teachers of Mathematics (NCTM). (1995) *Assessment Standards for School Mathematics*. Reston, VA: National Council of Teachers of Mathematics, p. 45.

Page 34: Allen, M. J. (2004). *Assessing Academic Progress in Higher Education*. Bolton, MA: Anker Publishing Company, Inc., p. 5.

Page 37: Adelman, C. (February 2006). *The Toolbox Revisited: Paths to Degree Completion from High School Through College*. Washington, DC: U.S. Department of Education, p. xix.

Page 40: The National Council on Education and the Disciplines (NCED). (2001). Steen, L. A. (Ed.) *Mathematics and Democracy: The Case of Quantitative Literacy*. Washington, DC: The Woodrow Wilson National Fellowship Foundation, p. 115.

Page 42: Mathematical Association of America. (2004). *Undergraduate Programs and Courses in the Mathematical Sciences: CUPM Curriculum Guide*. A Report by the Committee on the Undergraduate Program in Mathematics. Washington, DC: Mathematical Association of America, Executive Summary, p. 1.

Page 43: Hardy, G. H. (1941). *A Mathematician's Apology*. London, England: Cambridge University Press, p. 25.

Page 44: National Science Foundation. (July 1993). *Gaining the Competitive Edge: Critical Issues in Science and Engineering Technician Education*. A Report from a Workshop Sponsored by the National Science Foundation and the Federal Coordinating Council for Science, Engineering, and Technology. Arlington, VA: National Science Board, p. 19.

Page 45: Conference Board of the Mathematical Sciences. (2001). *The Mathematical Education of Teachers, Part I*. Washington, DC: Mathematical Association of America, p. 9.

Page 47: Rossi, H. (October 1996). *Notices of the American Mathematical Society*. 43(10), p. 1108.

Page 52: Mathematical Association of America. (2004). *The Curriculum Foundations Project, Voices of the Partner Disciplines*. Curriculum Renewal Across the First Two Years Subcommittee (CRAFTY). Washington, DC: Mathematical Association of America, p. 1

Page 53: National Research Council. (1999). Bransford, J. D., Brown, A. L., & Cocking, R. R. (Eds). *How People Learn: Brain, Mind, Experience, and School*. Washington, DC: National Academy Press, p. 152.

Page 56: National Science Foundation. (February 2006). *America's Pressing Challenge—Building a Stronger Foundation, A Companion to Science and Engineering Indicators 2006*. Arlington, VA: National Science Board, NSB 06-02, p. 6. Retrieved 3/1/2006 from http://www.nsf.gov/statistics/nsb0602.

Page 57: Simonson, M., Smaldina, S., Albright, M., & Zvacek, S. (2003). *Teaching and Teaching at a Distance*. Upper Saddle River, NJ: Pearson Education, Inc., p. 233.

Page 63: North Central Association of Colleges and Schools. (2005). *The Higher Learning Commission Handbook of Accreditation*, Third Edition. Chicago, IL: The Higher Learning Commission, p. 3-2-10.

Page 65: Jiménez, A. (2005). AMATYC/MAA Project ACCCESS Fellow, Pima Community College, Tucson, AZ. A quote solicited for this publication.

Page 67: American Mathematical Association of Two-Year Colleges (AMATYC). Neptune, C. (Ed.). (2001). *Opportunities for Excellence: Professionalism and the Two-Year College Mathematics Faculty*. Memphis, TN: American Mathematical Association of Two-Year Colleges, p. 48.

Page 68: University of Hawaii Community Colleges. (April 1990). *Procedures for Evaluation of Faculty*. Retrieved 3/31/2006 from http://www.honolulu.hawaii.edu/intranet/committees/FacDevCom/guidebk/reappt/fac-eval.htm.

Page 72: (Figure 5). ShareBuilder customer survey (February 2005). DDB, Seattle, WA.

Page 73: (Figure 6). Achieve, Inc. (2005). *Rising to the Challenge. Are High School Graduates Prepared for College & Work? Key findings from surveys by Hart Research Associates*. A PowerPoint presentation prepared by Achieve, Inc., Washington, DC. Retrieved 4/18/2006 from http://www.achieve.org/dstore.nsf/Lookup/poll/$file/poll.ppt, slide 15.

Page 75: Education Commission of the States. (February 2001). *Transfer and Articulation Policies*. Denver, CO. Retrieved 5/1/2006 from http://www.ecs.org/ecsmain.asp?page=/html/publications/home_publications.asp

Page 75: American Mathematical Association of Two-Year Colleges (AMATYC). (2003). Hovis, M. A., Kimball, R., & Peterson, J. (Eds.). *A Vision: Mathematics for the Emerging Technologies, A Report from the Project Technical Mathematics for Tomorrow: Recommendations and Exemplary Programs*, Memphis, TN: American Mathematical Association of Two-Year Colleges, p. 15.

Page 76: Business Roundtable. (July 2005). *Tapping America's Potential: The Education for Innovation Initiative*. Washington, DC: Business Roundtable, p. 14. Retrieved 3/30/2006 from www.businessroundtable.org/pdf/200508030001TAPfinalnb.pdf.

Page 76: Business–Higher Education Forum. (January 2005). *A Commitment to America's Future: Responding to the Crisis in Mathematics and Science Education*. Washington, DC: Business–Higher Education Forum, p. 32. www.bhef.com.

Page 77: Bumper sticker

Page 79: National Academy of Sciences, National Academy of Engineering, Institute of Medicine. (2005). *Rising Above the Gathering Storm: Energizing and Employing America for a Brighter Economic Future*. Committee on Prospering in the Global Economy of the 21st Century: An Agenda for American Science and Technology. Washington, DC: The National Academies, p. ES-8. Retrieved 4/18/2006 from http://fermat.nap.edu/books/0309100399/html/1.html
Full document: http://www.nap.edu/catalog/11463.html.

Page 80: Attributed to Albert Einstein.

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